Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 1

Exercise 1

Given the system

$u^2 - x - 2 u x^2 + x^3 - 3 u y - u x y + y^2 - 3 y^3 == 30$

$u - 3 x + 2 u^2 x + 2 x^2 + y^2 + 2 u y^2 == -82$

determine if it is possible to solve for variables x, y in terms of variable u arround the point $p = (x, y, u) = (-2, 2, -4)$. Compute if possible $\frac{\partial y}{\partial u}(-4)$.

1) $\frac{\partial y}{\partial u}(-4) = \frac{39}{238}$

2) $\frac{\partial y}{\partial u}(-4) = \frac{159}{952}$

3) $\frac{\partial y}{\partial u}(-4) = \frac{155}{952}$

4) $\frac{\partial y}{\partial u}(-4) = \frac{79}{476}$

5) $\frac{\partial y}{\partial u}(-4) = \frac{157}{952}$

Exercise 2

Compute $\int_D (3 x + y^3)\, dx dy$ for $D = \{3 y^{11} \le x^6 \le 5 y^{11},\ 6 x \le y^2 \le 7 x,\ x > 0,\ y > 0\}$

1) -1.87243×10^{31}

2) 5.61728×10^{31}

3) 6.68724×10^{31}

4) -2.13992×10^{31}

5) 2.6749×10^{31}

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) =
{-3 Cos[u] (2 + Cos[v]) + 3 (2 + Cos[v]) Sin[u] - 2 Sin[v],
3 Cos[u] (2 + Cos[v]) - 2 (2 + Cos[v]) Sin[u] + 2 Sin[v],
2 Cos[u] (2 + Cos[v]) - 2 (2 + Cos[v]) Sin[u] + Sin[v]} .

1) The maximum Gauss curvature is **1.****

2) The maximum Gauss curvature is **3.****

3) The maximum Gauss curvature is **2.****

4) The maximum Gauss curvature is **8.****

5) The maximum Gauss curvature is **5.****

Exercise 4

Compute the area of the domain whose boundary is the curve

r: [0, π] ----→ R^2
r(t) = { (t + 8) sin(2 t) (8 cos(10 t) + 8), (3 t + 7) sin(t) }

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 581.516 2) 1975.92 3) 1162.52 4) 2092.12

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-2)\,(x-1)\,x^2\,(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.005 by means of a Fourier series of order 8 .

1) u(2 , 0.005) = ***.5***

2) u(2 , 0.005) = ***.4***

3) u(2 , 0.005) = ***.7***

4) u(2 , 0.005) = ***.2***

5) u(2 , 0.005) = ***.3***

Further Mathematics - Degree in Engineering - 2025/2026
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Exercise 1

Given the function

$f(x, y, z) = -4x + x^2 + y^2 - 4z + z^2$ defined over the domain $D \equiv \frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {−4.86785, ?, −0.685154} and {2, 0, 2} is not a saddle point of f.

2) We have a maximum at {?, 0, 2} and {2, 0, 2} is not a local maximum of f.

3) We have a maximum at {−4.66785, ?, −0.185154} and {2, 0, 2} is not a saddle point of f.

4) We have a maximum at {−4.96785, 0.5, ?} and {2, 0, 2} is not a local maximum of f.

5) We have a maximum at {−5.06785, ?, −0.485154} and {2, 0, 2} is not a saddle point of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $7x + 3z == 2$ and the paraboloid $z == 2x^2 + 3y^2$ and the semiplanes $-3x - 7y \geq 0$ and $-4x - 3y \geq 0$.

1) 3.42577

2) 0.459809

3) 1.87329

4) 3.12007

5) 0.885923

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {5 Cos[u] Sin[v], −10 Cos[u] Sin[v] + Sin[u] Sin[v], 3 Cos[v] − 10 Cos[u] Sin[v] − 2 Sin[u] Sin[v]} .

1) The maximum Gauss curvature is **7.****

2) The maximum Gauss curvature is **4.****

3) The maximum Gauss curvature is **3.****

4) The maximum Gauss curvature is **9.****

5) The maximum Gauss curvature is **2.****

Exercise 4

Consider the vector field $F(x,y,z) = \{-3x - z + \text{Cos}[2y^2 + 2z^2],\ e^{x^2+z^2} - 9xz + 2yz,\ -2x + \text{Cos}[2y^2]\}$ and the surface

$$S \equiv \left(\frac{-8+x}{4}\right)^2 + \left(\frac{8+y}{7}\right)^2 + \left(\frac{7+z}{5}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −9969.32 2) −21 933.3 3) 7976.68 4) −45 861.3

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = (x-2)\,x^2\,(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.01 , by separation of variables by means of a Fourier series of order 9.

1) u(2 , 0.01) = ***.**7*

2) u(2 , 0.01) = ***.**4*

3) u(2 , 0.01) = ***.**0*

4) u(2 , 0.01) = ***.**1*

5) u(2 , 0.01) = ***.**3*

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 3

Exercise 1

Consider the domains $D_1 \equiv 22 - 16x + 7x^2 + 20y - 10xy + 5y^2 \leq 1$ and $D_2 \equiv 117 + 30x + 3x^2 - 18y + 2xy + 5y^2 \leq 100$.
Compute the distance between they two, $d(D_1,D_2)$, and the points where it is attained.

1) The distance between both domains is **7.****

2) The distance between both domains is **4.****

3) The distance between both domains is **3.****

4) The distance between both domains is **2.****

5) The distance between both domains is **1.****

Exercise 2

Compute the volume of the domain limited by the plane $7x + 5z == 1$ and the paraboloid $z == 8x^2 + 7y^2$ and the semiplanes $-6x - 8y \geq 0$ and $-2x + 9y \geq 0$.

1) 0.0166361

2) 0.0228941

3) 0.00475884

4) 0.0291099

5) 0.0445548

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{52 + 5u + 2u^2 + 19v + 2v^2, v, 26 + 2u + u^2 + 10v + v^2\}$.

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **8.****

3) The maximum Gauss curvature is **7.****

4) The maximum Gauss curvature is **0.****

5) The maximum Gauss curvature is **2.****

Exercise 4

Consider the vector field $F(x,y,z) = \{6xyz - \mathrm{Sin}[z^2],\ e^{2x^2+2z^2} - xz,\ 9z + 9yz - \mathrm{Sin}[2y^2]\}$ and the surface

$$S \equiv \left(\frac{5+x}{7}\right)^2 + \left(\frac{8+y}{2}\right)^2 + \left(\frac{z}{9}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 13 300.8 2) 79 802.8 3) −33 250.6 4) 26 601.2

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9(8 + 7t)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -8x & 0 \le x \le 1 \\ \frac{8x}{\pi-1} - \frac{8}{\pi-1} - 8 & 1 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.006 , by separation of variables by means of a Fourier series of order 11.

1) u(2 , 0.006) = **8.****

2) u(2 , 0.006) = **3.****

3) u(2 , 0.006) = **7.****

4) u(2 , 0.006) = **9.****

5) u(2 , 0.006) = **6.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 4

Exercise 1

Given the system

$3u^2 w + x y == 11$

$u - 2 u v x + y^2 == -53$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p=(x , y , u , v , w) = (-4 , 4 , 3 , -3 , 1). Compute if possible $\frac{\partial x}{\partial v}(3,-3,1)$.

1) $\frac{\partial x}{\partial v}(3,-3,1) = -\frac{10}{13}$

2) $\frac{\partial x}{\partial v}(3,-3,1) = -\frac{9}{13}$

3) $\frac{\partial x}{\partial v}(3,-3,1) = -\frac{8}{13}$

4) $\frac{\partial x}{\partial v}(3,-3,1) = -\frac{11}{13}$

5) $\frac{\partial x}{\partial v}(3,-3,1) = -\frac{12}{13}$

Exercise 2

Compute $\int_D (x+2y)\,dx\,dy$ for $D=\{9y^3 \le x^4 \le 12y^3,\ 9x^5 \le y^4 \le 10x^5,\ x>0,\ y>0\}$

1) -5.64508×10^{23}

2) 3.38705×10^{23}

3) 3.38705×10^{24}

4) 1.12902×10^{24}

5) -6.7741×10^{23}

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-2v+5(1+3v^2)\text{Cos}[u],\ -6(1+3v^2)\text{Cos}[u]+(1+3v^2)\text{Sin}[u],\ v-2(1+3v^2)\text{Cos}[u]\}$.

1) The maximum Gauss curvature is **8.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **0.****

4) The maximum Gauss curvature is **7.****

5) The maximum Gauss curvature is **2.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$
$r(t) = \{ (2t+8) \sin(2t) (\cos(16t)+6), (9t+4) \sin(t) \}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 3130.28 2) 2471.48 3) 1647.98 4) 1318.58

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-3)(x-1)x^2(x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1
and the moment t= 0.009 by means of a Fourier series of order 9 .

1) u(1 , 0.009) = ***.2***

2) u(1 , 0.009) = ***.1***

3) u(1 , 0.009) = ***.7***

4) u(1 , 0.009) = ***.0***

5) u(1 , 0.009) = ***.6***

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 5

Exercise 1

Given the system

$3u^3 - 2v + vy + 3vxy == -25$

$-uv^2 + 2x^2 - vxy + uz == 21$

$x^2 - 2uz == 17$

determine if it is possible to solve for variables x, y, z in terms of variables u, v arround the point p=(x, y, z, u, v)=(3, -2, 4, -1, 1). Compute if possible $\frac{\partial x}{\partial u}(-1, 1)$.

1) $\frac{\partial x}{\partial u}(-1, 1) = -\frac{2}{19}$

2) $\frac{\partial x}{\partial u}(-1, 1) = -\frac{17}{152}$

3) $\frac{\partial x}{\partial u}(-1, 1) = -\frac{7}{76}$

4) $\frac{\partial x}{\partial u}(-1, 1) = -\frac{13}{152}$

5) $\frac{\partial x}{\partial u}(-1, 1) = -\frac{15}{152}$

Exercise 2

Compute $\int_D (x + 3y)\,dx\,dy$ for $D = \{4y \le x^3 \le 9y,\ 9y^2 \le x^7 \le 12y^2,\ x > 0,\ y > 0\}$

1) 0.00101621

2) -1.19898

3) -1.49898

4) -1.09898

5) 0.401016

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = $\{-27u - 2u^2 - 2(50 - 3v + v^2), -u - v, -50 - 14u - u^2 + 2v - v^2\}$.

1) The maximum Gauss curvature is **5.****

2) The maximum Gauss curvature is **3.****

3) The maximum Gauss curvature is **4.****

4) The maximum Gauss curvature is **0.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$
$r(t) = \{(6t+4)\sin(2t)(9\cos(11t)+10), (3t+6)\sin(t)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 3245.04 2) 203.036 3) 2028.24 4) 1419.84

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t) = u(2,t) = 0 & 0 \le t \\ u(x,0) = -2(x-2)(x-1)x & 0 \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{2}{5}$
and the moment t = 0.004 by means of a Fourier series of order 11.

1) $u(\frac{2}{5}, 0.004)$ = ***.7***

2) $u(\frac{2}{5}, 0.004)$ = ***.2***

3) $u(\frac{2}{5}, 0.004)$ = ***.5***

4) $u(\frac{2}{5}, 0.004)$ = ***.0***

5) $u(\frac{2}{5}, 0.004)$ = ***.8***

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 6

Exercise 1

Given the function

$f(x,y,z) = -1 - 6x + x^2 - 4y + y^2 + 4z - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, -1.64077, 0.333744} and {3, 2, 2} is not a local maximum of f.

2) We have a maximum at {-2.36115, -1.44077, ?} and {3, 2, 2} is not a saddle point of f.

3) We have a maximum at {-2.26115, -1.74077, ?} and {3, 2, 2} is not a local minimum of f.

4) We have a maximum at {-2.36115, ?, 0.0337439} and {3, 2, 2} is not a saddle point of f.

5) We have a maximum at {3, 2, ?} and {3, 2, 2} is not a saddle point of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $6x + 4z == 1$ and the paraboloid $z == x^2 + y^2$.

1) 4.19288

2) 0.133912

3) 1.03697

4) 1.58111

5) 0.166407

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {Cos[u] Sin[v], 5 Cos[v] - 2 Cos[u] Sin[v], 15 Cos[v] - 2 Cos[u] Sin[v] - Sin[u] Sin[v]}.

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **0.****

3) The maximum Gauss curvature is **2.****

4) The maximum Gauss curvature is **8.****

5) The maximum Gauss curvature is **9.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r : [0,\pi] \longrightarrow R^2$

$r(t) = \{ (5t+9)\sin(2t)\,(3\cos(10t)+10),\ (t+4)\sin(t)\,(3\cos(10t)+10) \}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 13 261.3   2) 9282.96   3) 25 196.2   4) 1326.36

Exercise 5

$$\begin{cases} (1+t)\,\frac{\partial u}{\partial t}(x,t) = 9\,(1)\,\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{9x}{2} & 0 \le x \le 2 \\ 15 - 3x & 2 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 4
, t= 0.006 , by separation of variables by means of a Fourier series of order 8.

1) u(4 , 0.006) = **5.****

2) u(4 , 0.006) = **8.****

3) u(4 , 0.006) = **0.****

4) u(4 , 0.006) = **9.****

5) u(4 , 0.006) = **2.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 7

Exercise 1

Given the system

$-y\, z\, u_2 + 2\, z\, u_2\, u_4 == -144$
$3\, y\, u_1 - 3\, x\, y\, u_1 - u_2\, u_3 == 44$
$-2\, x\, y\, u_1 == 32$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 arround the point $p=(x, y, z, u_1, u_2, u_3, u_4) = (-4, 2, 3, 2, 4, 4, -5)$. Compute if possible $\frac{\partial x}{\partial u_4}(2, 4, 4, -5)$.

1) $\frac{\partial x}{\partial u_4}(2, 4, 4, -5) = 3$

2) $\frac{\partial x}{\partial u_4}(2, 4, 4, -5) = 4$

3) $\frac{\partial x}{\partial u_4}(2, 4, 4, -5) = 2$

4) $\frac{\partial x}{\partial u_4}(2, 4, 4, -5) = 0$

5) $\frac{\partial x}{\partial u_4}(2, 4, 4, -5) = 1$

Exercise 2

Compute $\int_D (x + x^2)\, dx\, dy$ for $D = \{2 x^9 y^2 \le 1 \le 8 x^9 y^2,\ 3 \le x^{14} y^3 \le 11,\ x > 0,\ y > 0\}$

1) 0.300076

2) 0.0000759227

3) -0.499924

4) 1.00008

5) 1.70008

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-5 + 4u - u^2 - 2v - v^2, v, 5 - 3u + u^2 + v^2\}$.

1) The maximum Gauss curvature is **8.****

2) The maximum Gauss curvature is **2.****

3) The maximum Gauss curvature is **1.****

4) The maximum Gauss curvature is **6.****

5) The maximum Gauss curvature is **4.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$
$r(t) = \{(8t + 9)\sin(2t)(5\cos(14t) + 9), (7t + 7)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 7149.63 2) 9056.03 3) 8102.83 4) 4766.63

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -2(x-3)(x-2)x^2(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2 and the moment t= 0.002 by means of a Fourier series of order 12 .

1) u(2 , 0.002) = ***.*1**

2) u(2 , 0.002) = ***.*2**

3) u(2 , 0.002) = ***.*6**

4) u(2 , 0.002) = ***.*5**

5) u(2 , 0.002) = ***.*4**

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 8

Exercise 1

Given the function

$f(x, y, z) = 2 - 4x + x^2 - 2y + y^2 - 2z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {2, ?, 1} and {-1, 0, -1} is not a saddle point of f.

2) We have a minimum at {1.2, 1.8, ?} and {2, 1, 1} is not a local minimum of f.

3) We have a minimum at {1.4, 1.8, ?} and {2, 1, 1} is not a local maximum of f.

4) We have a minimum at {2.8, 0.8, ?} and {2, 1, 1} is not a local maximum of f.

5) We have a minimum at {?, 1.4, 0.2} and {2, 1, 1} is not a local minimum of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $7x + 3z == 6$ and the paraboloid $z == 4x^2 + 4y^2$.

1) 2.76208

2) 4.05449

3) 10.2829

4) 0.936391

5) 2.15077

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{(1 + v^2) \text{Cos}[u], (1 + v^2) \text{Sin}[u], v + 2(1 + v^2) \text{Cos}[u] - 2(1 + v^2) \text{Sin}[u]\}$.

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **1.****

3) The maximum Gauss curvature is **7.****

4) The maximum Gauss curvature is **5.****

5) The maximum Gauss curvature is **2.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \dashrightarrow R^2$

$r(t)=\{(9t+5)\sin(2t)\,(2\cos(16t)+2),\ (t+7)\sin(t)\,(2\cos(16t)+2)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 798.106 2) 2128.11 3) 1330.11 4) 1197.11

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=2\,(x-1)\left(x-\frac{3}{5}\right)x^2 & 0\le x\le 1 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} 70x & 0\le x\le \frac{1}{10} \\ \frac{37}{4}-\frac{45x}{2} & \frac{1}{10}\le x\le \frac{1}{2} \\ 4x-4 & \frac{1}{2}\le x\le 1 \end{cases} & 0.\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=\frac{4}{5}$
and the moment $t=0.006$ by means of a Fourier series of order 8 .

1) $u\left(\frac{4}{5}, 0.006\right)$ = ***.*8**

2) $u\left(\frac{4}{5}, 0.006\right)$ = ***.*5**

3) $u\left(\frac{4}{5}, 0.006\right)$ = ***.*1**

4) $u\left(\frac{4}{5}, 0.006\right)$ = ***.*4**

5) $u\left(\frac{4}{5}, 0.006\right)$ = ***.*3**

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 9

Exercise 1

Given the system

$-x\,y\,u_4 - 3\,y^2\,u_4 - u_1\,u_4 == 15$
$-3\,x^2\,y == -96$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (4, 2, -5, 2, -4, -1)$. Compute if possible the approximate values of (x, y) for $(u_1, u_2, u_3, u_4) = (-4.8, 1.8, -3.9, -1.3)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (4.63571, 2.16429)$

2) $(x, y) \simeq (4.63571, 1.26429)$

3) $(x, y) \simeq (4.33571, 1.66429)$

4) $(x, y) \simeq (3.83571, 1.46429)$

5) $(x, y) \simeq (4.23571, 1.36429)$

Exercise 2

Compute the volume of the domain limited by the plane $3x + 4z == 5$ and the paraboloid $z == 8x^2 + 7y^2$ and the semiplanes $4x + 2y \geq 0$ and $5x - 4y \geq 0$.

1) 0.0781081

2) 0.19224

3) 0.383073

4) 0.211608

5) 0.106302

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {3 Cos[u] (3 + Cos[v]) + 7 (3 + Cos[v]) Sin[u], 2 Cos[u] (3 + Cos[v]) + 5 (3 + Cos[v]) Sin[u], Cos[u] (3 + Cos[v]) + 2 (3 + Cos[v]) Sin[u] + Sin[v]} .

1) The maximum Gauss curvature is **8.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **0.****

4) The maximum Gauss curvature is **4.****

5) The maximum Gauss curvature is **9.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{-9 + e^{-y^2+2z^2},\ 7z + \mathrm{Cos}[z^2],\ e^{2y^2} - x + 9y\right\}$ and the surface

$$S \equiv \left(\frac{5+x}{6}\right)^2 + \left(\frac{-4+y}{8}\right)^2 + \left(\frac{6+z}{2}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 1.7 2) 2.4 3) 2.7 4) 0.

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -4x & 0 \le x \le 1 \\ x-5 & 1 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -\frac{5x}{2} & 0 \le x \le 2 \\ 6x-17 & 2 \le x \le 3 \\ \frac{5}{2} - \frac{x}{2} & 3 \le x \le 5 \end{cases} & 0. \le x \le 5 \\ 0 & \mathrm{True} \end{cases}$$

Compute the position of the string at $x = 1$
and the moment $t = 0.005$ by means of a Fourier series of order 8 .

1) u (1 , 0.005) = **3.****

2) u (1 , 0.005) = **9.****

3) u (1 , 0.005) = **6.****

4) u (1 , 0.005) = **0.****

5) u (1 , 0.005) = **7.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 10

Exercise 1

Consider the domains $D_1 \equiv 180 - 8x + 4x^2 - 54y + 10xy + 9y^2 \leq 1$ and $D_2 \equiv 280 - 50x + 5x^2 - 32y - 6xy + 8y^2 \leq 1000$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (* , ***.**9*)

2) The point of D_1 closest to D_2 is (* , ***.**7*)

3) The point of D_1 closest to D_2 is (* , ***.**1*)

4) The point of D_1 closest to D_2 is (* , ***.**3*)

5) The point of D_1 closest to D_2 is (* , ***.**6*)

Exercise 2

Compute the volume of the domain limited by the plane $8x + 9z == 3$ and the paraboloid $z == 5x^2 + y^2$ and the semiplanes $-x - 3y \geq 0$ and $-7x - 4y \geq 0$.

1) 0.0573399

2) 0.238399

3) 0.0498628

4) 0.1643

5) 0.0914296

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{v + (1 + 2v^2)\text{Cos}[u] + 4(1 + 2v^2)\text{Sin}[u], (1 + 2v^2)\text{Sin}[u], v\}$.

1) The maximum Gauss curvature is **2.****

2) The maximum Gauss curvature is **1.****

3) The maximum Gauss curvature is **8.****

4) The maximum Gauss curvature is **3.****

5) The maximum Gauss curvature is **0.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{e^{-2y^2+2z^2} + 6xz + yz,\ -8y + \mathrm{Sin}[x^2],\ e^{-x^2} + 9xz + 9xyz\right\}$ and the surface

$$S \equiv \left(\frac{-4+x}{1}\right)^2 + \left(\frac{-2+y}{9}\right)^2 + \left(\frac{-6+z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 20 508.3   2) −53 320.5   3) 6152.72   4) −59 472.9

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0 & 0 \le t \\ u(x,0) = -3(x-1)^2\left(x-\frac{4}{5}\right)\left(x-\frac{3}{10}\right)x^2 & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{1}{10}$ and the moment $t = 0.009$ by means of a Fourier series of order 12 .

1) $u\left(\frac{1}{10}, 0.009\right)$ = ***.****6

2) $u\left(\frac{1}{10}, 0.009\right)$ = ***.****9

3) $u\left(\frac{1}{10}, 0.009\right)$ = ***.****3

4) $u\left(\frac{1}{10}, 0.009\right)$ = ***.****7

5) $u\left(\frac{1}{10}, 0.009\right)$ = ***.****5

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 11

Exercise 1

Given the system

$u\,v^2 - 3\,u\,x_2\,x_3 == 24$

$-3\,u\,x_1\,x_3 + 3\,u\,x_3^2 == -24$

$-2\,u\,v\,x_2 + 2\,u^2\,x_3 - x_1\,x_3 - 3\,v^2\,x_4 == -52$

$3\,u^2\,x_2 - x_2\,x_3\,x_4 == 24$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (-5, 1, -4, 3, 2, 0)$. Compute if possible $\frac{\partial x_4}{\partial u}(2, 0)$.

1) $\frac{\partial x_4}{\partial u}(2, 0) = -\frac{45}{32}$

2) $\frac{\partial x_4}{\partial u}(2, 0) = -\frac{11}{8}$

3) $\frac{\partial x_4}{\partial u}(2, 0) = -\frac{43}{32}$

4) $\frac{\partial x_4}{\partial u}(2, 0) = -\frac{21}{16}$

5) $\frac{\partial x_4}{\partial u}(2, 0) = -\frac{41}{32}$

Exercise 2

Compute $\int_D (y^5)\,dx\,dy$ for $D = \{x\,y^3 \leq 1 \leq 9\,x\,y^3,\ 8 \leq x^2\,y^5 \leq 10,\ x > 0,\ y > 0\}$

1) -1.99995

2) 0.0000453869

3) -0.0999546

4) 1.40005

5) -1.99995

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) =$
$\{(1+v^2)\,\text{Cos}[u],\ 3v - 2(1+v^2)\,\text{Cos}[u] + (1+v^2)\,\text{Sin}[u],\ v + (1+v^2)\,\text{Cos}[u]\}$.

1) The maximum Gauss curvature is **0.****

2) The maximum Gauss curvature is **3.****

3) The maximum Gauss curvature is **4.****

4) The maximum Gauss curvature is **9.****

5) The maximum Gauss curvature is **7.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$
$r(t) = \{(3t+1)\sin(2t)(9\cos(19t)+10),\ (2t+8)\sin(t)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 1312.88   2) 875.381   3) 1225.38   4) 525.381

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 3x & 0 \le x \le 3 \\ -\frac{9x}{\pi-3} + \frac{27}{\pi-3} + 9 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$
and the moment $t = 0.004$ by means of a Fourier series of order 8 .

1) u(1 , 0.004) = **5.****

2) u(1 , 0.004) = **8.****

3) u(1 , 0.004) = **3.****

4) u(1 , 0.004) = **1.****

5) u(1 , 0.004) = **4.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 12

Exercise 1

Given the system

$2\,u\,w\,y - 2\,u\,y\,z == -180$

$-2\,x\,z^2 == 24$

$-x\,z == 6$

determine if it is possible to solve for variables x, y, z in terms of variables u, v, w arround the point $p=(x, y, z, u, v, w)=(-3, 5, 2, 3, 3, -4)$. Compute if possible $\frac{\partial y}{\partial v}(3, 3, -4)$.

1) $\frac{\partial y}{\partial v}(3, 3, -4) = 0$

2) $\frac{\partial y}{\partial v}(3, 3, -4) = 4$

3) $\frac{\partial y}{\partial v}(3, 3, -4) = 3$

4) $\frac{\partial y}{\partial v}(3, 3, -4) = 2$

5) $\frac{\partial y}{\partial v}(3, 3, -4) = 1$

Exercise 2

Compute $\int_D (2\,x\,y)\,dx\,dy$ for $D=\{7\,x^8 \le y^5 \le 11\,x^8,\ 6\,x^{21} \le y^{13} \le 7\,x^{21},\ x > 0,\ y > 0\}$

1) 4.24965×10^{47}

2) 8.07434×10^{47}

3) 5.52455×10^{47}

4) 9.7742×10^{47}

5) 1.10491×10^{48}

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) =
{-2 Cos[v] + 5 Cos[u] Sin[v], 4 Cos[v] + 5 Cos[u] Sin[v] + 4 Sin[u] Sin[v], 5 Cos[u] Sin[v]} .

1) The maximum Gauss curvature is **1.****

2) The maximum Gauss curvature is **4.****

3) The maximum Gauss curvature is **9.****

4) The maximum Gauss curvature is **7.****

5) The maximum Gauss curvature is **5.****

Exercise 4

Compute the area of the domain whose boundary is the curve

r: [0,π] ----→ R^2
r(t) = {(9 t + 2) sin(2 t) (7 cos(19 t) + 9), (4 t + 4) sin(t)}

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 1481.94 2) 2116.74 3) 2539.94 4) 1905.14

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-2)(x-1)x^2(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1
and the moment t= 0.008 by means of a Fourier series of order 11 .

1) u(1 , 0.008) = ***.*5**

2) u(1 , 0.008) = ***.*4**

3) u(1 , 0.008) = ***.*6**

4) u(1 , 0.008) = ***.*1**

5) u(1 , 0.008) = ***.*8**

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 13

Exercise 1

Consider the domain $D_1 \equiv 5x^2 + 2xy + 5y^2 = 1$ and the point $q = (-5, -5)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is ∗∗∗.∗∗8∗

2) The distance between D_1 and q is ∗∗∗.∗∗3∗

3) The distance between D_1 and q is ∗∗∗.∗∗2∗

4) The distance between D_1 and q is ∗∗∗.∗∗9∗

5) The distance between D_1 and q is ∗∗∗.∗∗0∗

Exercise 2

Compute $\int_D (y z^2)\, dx\, dy\, dz$ for $D = \{9 z^8 \le x^7 y^3 \le 15 z^8,\ 8 y^5 \le x^7 z^4 \le 10 y^5,\ 6 x^4 z^2 \le 1 \le 10 x^4 z^2,\ x > 0,\ y > 0,\ z > 0\}$

1) 1.61451×10^{-6}

2) -1.2

3) 0.900002

4) $1.$

5) 0.100002

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-3u - 2v,\ 2u + v,\ -2(41 + 8u + u^2 - 10v + v^2)\}$.

1) The maximum Gauss curvature is ∗∗3.∗∗∗∗

2) The maximum Gauss curvature is ∗∗4.∗∗∗∗

3) The maximum Gauss curvature is ∗∗8.∗∗∗∗

4) The maximum Gauss curvature is ∗∗7.∗∗∗∗

5) The maximum Gauss curvature is ∗∗6.∗∗∗∗

Exercise 4

Consider the vector field $F(x,y,z) = \left\{e^{2 y^2} - 5 x,\ 4 y + \text{Sin}\left[2 x^2 + 2 z^2\right],\ e^{-x^2+2 y^2} - 9 y z\right\}$ and the surface

$$S \equiv \left(\frac{9+x}{5}\right)^2 + \left(\frac{6+y}{7}\right)^2 + \left(\frac{9+z}{1}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 35 742.2 2) 25 641.2 3) −16 316.8 4) 7770.21

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -90 x & 0 \le x \le \frac{1}{10} \\ 110 x - 20 & \frac{1}{10} \le x \le \frac{1}{5} \\ \frac{5}{2} - \frac{5 x}{2} & \frac{1}{5} \le x \le 1 \end{cases} & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = \frac{1}{5}$, $t = 0.008$, by separation of variables by means of a Fourier series of order 8.

1) $u\left(\frac{1}{5}, 0.008\right)$ = ***.1***

2) $u\left(\frac{1}{5}, 0.008\right)$ = ***.5***

3) $u\left(\frac{1}{5}, 0.008\right)$ = ***.2***

4) $u\left(\frac{1}{5}, 0.008\right)$ = ***.8***

5) $u\left(\frac{1}{5}, 0.008\right)$ = ***.3***

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 14

Exercise 1

Consider the domain $D_1 \equiv 8x^2 - 2xy + 7y^2 = 1$ and the point $q = (-7, 0)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is **5.****

2) The distance between D_1 and q is **6.****

3) The distance between D_1 and q is **0.****

4) The distance between D_1 and q is **7.****

5) The distance between D_1 and q is **4.****

Exercise 2

Compute $\int_D (2yz)\,dx\,dy\,dz$ for $D =$
$\{4z^6 \le x^7 y^8 \le 6z^6,\ 5x^8 z^4 \le y^6 \le 11x^8 z^4,\ 2x^7 y^4 \le z^2 \le 6x^7 y^4,\ x > 0,\ y > 0,\ z > 0\}$

1) −0.499945

2) −0.0999449

3) 1.00006

4) 0.0000550777

5) −1.89994

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) =$
$\{\cos[u]\,(3 + 2\cos[v]) - 2\,(3 + 2\cos[v])\,\sin[u] + 8\sin[v],$
$(3 + 2\cos[v])\,\sin[u] - 2\sin[v],\ 2\,(3 + 2\cos[v])\,\sin[u] - 3\sin[v]\}$.

1) The maximum Gauss curvature is **4.****

2) The maximum Gauss curvature is **2.****

3) The maximum Gauss curvature is **1.****

4) The maximum Gauss curvature is **9.****

5) The maximum Gauss curvature is **3.****

Exercise 4

Consider the vector field $F(x,y,z) = \{-7z + \cos[2y^2], -7xz + 3yz + \cos[x^2 + z^2], \sin[2x^2 - 2y^2]\}$ and the surface

$$S \equiv \left(\frac{8+x}{9}\right)^2 + \left(\frac{7+y}{1}\right)^2 + \left(\frac{2+z}{7}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 4752.64 2) −5543.36 3) −1583.36 4) −6810.56

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \mathrm{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = (x-2)\, x^2\, (x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = 1$, $t = 0.003$, by separation of variables by means of a Fourier series of order 8.

1) u(1 , 0.003) = **2.****

2) u(1 , 0.003) = **4.****

3) u(1 , 0.003) = **7.****

4) u(1 , 0.003) = **8.****

5) u(1 , 0.003) = **0.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 15

Exercise 1

Given the function

$f(x,y,z) = -7 - 4x + x^2 + 2y - y^2 + 6z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-1.98762, -0.101428, ?} and {2, 1, 3} is not a local minimum of f.

2) We have a maximum at {-3.50904, -0.25357, ?} and {2, 1, 3} is not a local minimum of f.

3) We have a maximum at {-3.50904, ?, 2.28213} and {2, 1, 3} is not a local minimum of f.

4) We have a maximum at {-2.74833, 0.507141, ?} and {2, 1, 3} is not a local maximum of f.

5) We have a maximum at {?, 1, 3} and {2, 1, 3} is not a local maximum of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $7x + 8z == 5$ and the paraboloid $z == 5x^2 + 5y^2$.

1) 0.138212

2) 0.533905

3) 0.249978

4) 0.432715

5) 0.118301

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-50 + 9u - u^2 + 11v - v^2, -50 + 9u - u^2 + 10v - v^2, u\}$.

1) The maximum Gauss curvature is **9.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **0.****

4) The maximum Gauss curvature is **4.****

5) The maximum Gauss curvature is **2.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$

$r(t) = \{(4t+3)\sin(2t)(7\cos(2t)+9),\ (3t+5)\sin(t)(7\cos(2t)+9)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 5839.79 2) 1668.79 3) 4588.49 4) 4171.39

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -4x & 0 \le x \le 1 \\ 2x-6 & 1 \le x \le 3 \end{cases} & 0 \le x \le 3 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} 7x & 0 \le x \le 1 \\ \frac{21}{2} - \frac{7x}{2} & 1 \le x \le 3 \end{cases} & 0. \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 2 and the moment t= 0.003 by means of a Fourier series of order 12 .

1) u(2 , 0.003) = **3.****

2) u(2 , 0.003) = **6.****

3) u(2 , 0.003) = **1.****

4) u(2 , 0.003) = **7.****

5) u(2 , 0.003) = **4.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 16

Exercise 1

Consider the domain $D_1 \equiv 8x^2 - 2xy + 5y^2 = 1$ and the point $q = (1, 4)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \bigcup \{q\}$ is **3.****

2) The diameter of $D_1 \bigcup \{q\}$ is **4.****

3) The diameter of $D_1 \bigcup \{q\}$ is **0.****

4) The diameter of $D_1 \bigcup \{q\}$ is **6.****

5) The diameter of $D_1 \bigcup \{q\}$ is **2.****

Exercise 2

Compute $\int_D (2 y z^3)\, dx\, dy\, dz$ for $D = \{2 y^9 \le x z^8 \le 8 y^9,\ 9 x^6 \le y^7 z^3 \le 12 x^6,\ 9 x \le y^8 z^9 \le 10 x,\ x > 0,\ y > 0,\ z > 0\}$

1) −1.8998

2) 0.000203129

3) −1.2998

4) −1.3998

5) −0.399797

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-19u - 2u^2 - 2(89 - 16v + v^2),\ -356 - 40u - 4u^2 + 65v - 4v^2,\ 2(89 + 10u + u^2 - 16v + v^2)\}$.

1) The maximum Gauss curvature is **9.****

2) The maximum Gauss curvature is **5.****

3) The maximum Gauss curvature is **7.****

4) The maximum Gauss curvature is **4.****

5) The maximum Gauss curvature is **6.****

Exercise 4

Consider the vector field $F(x,y,z) =$
$\{4 + 4x + \text{Cos}[y^2 + z^2],\ 9z - \text{Sin}[z^2],\ 2 + 7y - \text{Sin}[2x^2 + 2y^2]\}$ and the surface

$S \equiv \left(\frac{-5 + x}{5}\right)^2 + \left(\frac{8 + y}{2}\right)^2 + \left(\frac{6 + z}{2}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 335.103   2) −770.397   3) 1239.6   4) −234.397

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = -3\,(x-1)\left(x - \frac{2}{5}\right) x & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{7}{10}$

and the moment $t = 0.003$ by means of a Fourier series of order 8 .

1) $u\left(\frac{7}{10}, 0.003\right)$ = ***.4***

2) $u\left(\frac{7}{10}, 0.003\right)$ = ***.1***

3) $u\left(\frac{7}{10}, 0.003\right)$ = ***.8***

4) $u\left(\frac{7}{10}, 0.003\right)$ = ***.9***

5) $u\left(\frac{7}{10}, 0.003\right)$ = ***.6***

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 17

Exercise 1

Given the system

$-u^2 - x^3 - 2y - 3u^2 y + 3uxy - 2y^3 == 5$

$-3 - 3u + u^2 - u^3 - 2ux + 2y - 2uy - 2u^2 y + uxy + 3uy^2 + 2xy^2 + 3y^3 == -4$

determine if it is possible to solve for variables x , y in terms of variable u arround the point $p = (x, y, u) = (1, -1, 2)$. Compute if possible $\frac{\partial x}{\partial u}(2)$.

1) $\frac{\partial x}{\partial u}(2) = \frac{91}{79}$

2) $\frac{\partial x}{\partial u}(2) = \frac{90}{79}$

3) $\frac{\partial x}{\partial u}(2) = \frac{89}{79}$

4) $\frac{\partial x}{\partial u}(2) = \frac{92}{79}$

5) $\frac{\partial x}{\partial u}(2) = \frac{93}{79}$

Exercise 2

Compute $\int_D (x + y^2)\, dx dy$ for $D = \{4 \le x^{49} y^{132} \le 6,\ 6x^{13} y^{35} \le 1 \le 10 x^{13} y^{35},\ x > 0,\ y > 0\}$

1) 2.77203×10^{16}

2) 2.94528×10^{16}

3) 2.59878×10^{16}

4) 1.21276×10^{16}

5) 1.73252×10^{16}

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-904 + 113u - 8u^2 + 130v - 8v^2, -169u + 12u^2 + 3(452 - 65v + 4v^2), -1130 + 140u - 10u^2 + 163v - 10v^2\}$.

1) The maximum Gauss curvature is **3.****

2) The maximum Gauss curvature is **4.****

3) The maximum Gauss curvature is **7.****

4) The maximum Gauss curvature is **0.****

5) The maximum Gauss curvature is **6.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \longrightarrow R^2$
$r(t)=\{(t+1)\sin(2t)\,(2\cos(10t)+9),\ (7t+3)\sin(t)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 447.849 2) 850.149 3) 671.349 4) 537.249

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=(x-1)\,x\,(x-\pi)^2 & 0\le x\le\pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=1$
and the moment $t=0.008$ by means of a Fourier series of order 12.

1) u(1 , 0.008) = ***.**1*

2) u(1 , 0.008) = ***.**6*

3) u(1 , 0.008) = ***.**9*

4) u(1 , 0.008) = ***.**7*

5) u(1 , 0.008) = ***.**8*

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Exercise 1

Consider the domain $D_1 \equiv 5x^2 + 6xy + 3y^2 = 1$ and the point $q = (2, 8)$. Compute the diameter or largest distance between they two, $diam(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \bigcup \{q\}$ is ***.4***

2) The diameter of $D_1 \bigcup \{q\}$ is ***.8***

3) The diameter of $D_1 \bigcup \{q\}$ is ***.5***

4) The diameter of $D_1 \bigcup \{q\}$ is ***.3***

5) The diameter of $D_1 \bigcup \{q\}$ is ***.0***

Exercise 2

Compute $\int_D (x + 2y)\, dx dy dz$ for $D = \{7y^7 z^3 \le x^9 \le 11 y^7 z^3,\ 9z^5 \le x^4 y^9 \le 12 z^5,\ 6z^3 \le x^6 y^3 \le 12 z^3,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.0027676

2) −0.297232

3) −1.19723

4) 1.00277

5) 1.80277

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-5\, Cos[u]\, (4 + 2\, Cos[v]) + 6\, (4 + 2\, Cos[v])\, Sin[u] + 3\, Sin[v],\ (4 + 2\, Cos[v])\, Sin[u],\ -2\, Cos[u]\, (4 + 2\, Cos[v]) + 2\, (4 + 2\, Cos[v])\, Sin[u] + Sin[v]\}$.

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **1.****

3) The maximum Gauss curvature is **8.****

4) The maximum Gauss curvature is **4.****

5) The maximum Gauss curvature is **0.****

Exercise 4

Consider the vector field $F(x,y,z) =$
$\left\{e^{z^2} - 3z - 6xyz,\ e^{2z^2} + 6x - 3z,\ y + \mathrm{Sin}\left[2x^2 - y^2\right]\right\}$ and the surface

$S \equiv \left(\frac{x}{7}\right)^2 + \left(\frac{5+y}{7}\right)^2 + \left(\frac{5+z}{9}\right)^2 = 1$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) -1.13606×10^6 2) $-277088.$ 3) -55417.3 4) $-665013.$

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 3x & 0 \le x \le 1 \\ -\frac{3x}{\pi-1} + \frac{3}{\pi-1} + 3 & 1 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$
and the moment $t = 0.008$ by means of a Fourier series of order 9 .

1) u (1 , 0.008) = **0.****

2) u (1 , 0.008) = **2.****

3) u (1 , 0.008) = **9.****

4) u (1 , 0.008) = **7.****

5) u (1 , 0.008) = **1.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 19

Exercise 1

Given the function

$f(x,y,z) = 1 + 2x - x^2 + 6y - y^2 - 6z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {1, ?, 3} and {1, 3, 3} is not a local maximum of f.

2) We have a minimum at {-2.3548, ?, 1.43928} and {1, 3, 3} is not a local maximum of f.

3) We have a minimum at {-1.63516, -0.661957, ?} and {1, 3, 3} is not a saddle point of f.

4) We have a minimum at {?, -1.26165, 1.1994} and {1, 3, 3} is not a local minimum of f.

5) We have a minimum at {-2.47474, -1.38159, ?} and {1, 3, 3} is not a saddle point of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $5x + 6z == 2$ and the paraboloid $z == 9x^2 + 9y^2$.

1) 0.104368

2) 0.0986875

3) 0.0530323

4) 0.021702

5) 0.0333949

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {-2 Cos[v] - 3 Cos[u] Sin[v] + 12 Sin[u] Sin[v], 3 Sin[u] Sin[v], 2 Cos[v] + 6 Cos[u] Sin[v] - 15 Sin[u] Sin[v]} .

1) The maximum Gauss curvature is **2.****

2) The maximum Gauss curvature is **7.****

3) The maximum Gauss curvature is **6.****

4) The maximum Gauss curvature is **5.****

5) The maximum Gauss curvature is **3.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \longrightarrow R^2$
$r(t) = \{ (2t+8)\sin(2t)\,(5\cos(6t)+7),\ (7t+4)\sin(t)\,(5\cos(6t)+7) \}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 24 058.7   2) 9906.75   3) 14 152.3   4) 7076.35

Exercise 5

$$\begin{cases} (1+2t+2t^2)\frac{\partial u}{\partial t}(x,t) = 9(2+4t)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -3x & 0\le x\le 2 \\ \frac{6x}{\pi-2}-\frac{12}{\pi-2}-6 & 2\le x\le\pi \end{cases} & 0\le x\le\pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 1
, t= 0.006 , by separation of variables by means of a Fourier series of order 10.

1) u(1 , 0.006) = **7.****

2) u(1 , 0.006) = **2.****

3) u(1 , 0.006) = **5.****

4) u(1 , 0.006) = **9.****

5) u(1 , 0.006) = **1.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 20

Exercise 1

Consider the domain $D_1 \equiv 7x^2 + 8xy + 5y^2 = 1$ and the point $q = (-5, 3)$. Compute the diameter or largest distance between they two, diam(D_1, q), and the points of D_1 where it is attained.

1) The diameter of $D_1 \bigcup \{q\}$ is **1.****

2) The diameter of $D_1 \bigcup \{q\}$ is **8.****

3) The diameter of $D_1 \bigcup \{q\}$ is **4.****

4) The diameter of $D_1 \bigcup \{q\}$ is **5.****

5) The diameter of $D_1 \bigcup \{q\}$ is **6.****

Exercise 2

Compute $\int_D (3x + z^3)\,dx\,dy\,dz$ for $D =$
$\{5x^3 \le y^2 z^3 \le 7x^3,\ 8 \le x^6 y^4 z^9 \le 17,\ 4x^3 z^3 \le y^5 \le 10x^3 z^3,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.00571401

2) 0.805714

3) -0.594286

4) 1.30571

5) -1.59429

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{8\,\text{Cos}[v] + 25\,\text{Cos}[u]\,\text{Sin}[v] + 8\,\text{Sin}[u]\,\text{Sin}[v],\ -2\,\text{Cos}[v] - 5\,\text{Cos}[u]\,\text{Sin}[v],\ 4\,\text{Cos}[v] + 10\,\text{Cos}[u]\,\text{Sin}[v] + 4\,\text{Sin}[u]\,\text{Sin}[v]\}$.

1) The maximum Gauss curvature is **2.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **8.****

4) The maximum Gauss curvature is **9.****

5) The maximum Gauss curvature is **1.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{e^{2y^2-z^2} + 5xy,\ e^{-2x^2+z^2},\ 8 - 5xyz + \text{Cos}\left[x^2 - 2y^2\right]\right\}$ and the surface

$S \equiv \left(\frac{3+x}{4}\right)^2 + \left(\frac{-4+y}{7}\right)^2 + \left(\frac{-5+z}{5}\right)^2 = 1$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 103 211.   2) 117 285.   3) 4691.85   4) 46 914.5

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = 2(x-3)(x-2)x(x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.009$, by separation of variables by means of a Fourier series of order 12.

1) u(1 , 0.009) = *2*.****

2) u(1 , 0.009) = *0*.****

3) u(1 , 0.009) = *1*.****

4) u(1 , 0.009) = *4*.****

5) u(1 , 0.009) = *5*.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 21

Exercise 1

Consider the domains $D_1 \equiv 136 - 52x + 5x^2 + 64y - 12xy + 8y^2 \leq 1$ and $D_2 \equiv 568 - 88x + 6x^2 + 76y - 2xy + 4y^2 \leq 200$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (* , ***.**2*)

2) The point of D_1 closest to D_2 is (* , ***.**4*)

3) The point of D_1 closest to D_2 is (* , ***.**7*)

4) The point of D_1 closest to D_2 is (* , ***.**1*)

5) The point of D_1 closest to D_2 is (* , ***.**5*)

Exercise 2

Compute the volume of the domain limited by the plane $3x + z == 5$ and the paraboloid $z == 2x^2 + 9y^2$ and the semiplanes $-8x + 6y \geq 0$ and $-x + 6y \geq 0$.

1) 0.345703

2) 24.3479

3) 34.707

4) 8.01305

5) 3.71008

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{50 - 19u + 2u^2 + 2v^2, 100 - 39u + 4u^2 + v + 4v^2, -125 + 50u - 5u^2 - 2v - 5v^2\}$.

1) The maximum Gauss curvature is **9.****

2) The maximum Gauss curvature is **3.****

3) The maximum Gauss curvature is **4.****

4) The maximum Gauss curvature is **5.****

5) The maximum Gauss curvature is **7.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{-8+\mathrm{Sin}\left[y^2-z^2\right],\ -3\,x\,y+\mathrm{Sin}\left[x^2-z^2\right],\ e^{-x^2-y^2}\right\}$ and the surface

$$S\equiv \left(\frac{8+x}{8}\right)^2+\left(\frac{9+y}{2}\right)^2+\left(\frac{-7+z}{6}\right)^2=1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 9650.97   2) 10616.   3) −9649.03   4) −16404.

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=2\,(x-3)\,(x-1)\,x\,(x-\pi)^2 & 0\le x\le \pi \\ \frac{\partial}{\partial t}u(x,0)=(x-2)\,(x-1)\,x\,(x-\pi)^2 & 0.\le x\le \pi \\ 0 & \mathrm{True} \end{cases}$$

Compute the position of the string at x= 2
and the moment t= 0.007 by means of a Fourier series of order 9 .

1) u(2 , 0.007) = **1.****

2) u(2 , 0.007) = **6.****

3) u(2 , 0.007) = **3.****

4) u(2 , 0.007) = **5.****

5) u(2 , 0.007) = **7.****

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Exercise 1

Given the system

$-2\, x_2\, x_4^2 == 64$
$-3\, v\, x_3\, x_4 == 144$
$-3\, w\, x_1\, x_2 - 3\, w\, x_3^2 - 3\, u\, x_3\, x_4 == 120$
$2\, v\, x_2\, x_3 == 48$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v, w arround the point $p = (x_1, x_2, x_3, x_4, u, v, w) = (1, -2, 3, 4, -1, -4, -4)$. Compute if possible $\frac{\partial x_1}{\partial u}(-1, -4, -4)$.

1) $\frac{\partial x_1}{\partial u}(-1, -4, -4) = -\frac{1}{2}$

2) $\frac{\partial x_1}{\partial u}(-1, -4, -4) = 0$

3) $\frac{\partial x_1}{\partial u}(-1, -4, -4) = \frac{1}{2}$

4) $\frac{\partial x_1}{\partial u}(-1, -4, -4) = -\frac{3}{2}$

5) $\frac{\partial x_1}{\partial u}(-1, -4, -4) = -1$

Exercise 2

Compute $\int_D (x + y)\, dx\, dy$ for $D = \{9 x^3 \le y^2 \le 15 x^3,\ 8 x^2 \le y \le 13 x^2,\ x > 0,\ y > 0\}$

1) -0.498253

2) -1.59825

3) 0.00174743

4) 1.70175

5) -1.79825

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = $\{4v + (1+v^2)\,\text{Cos}[u],\ v - (1+v^2)\,\text{Cos}[u] + (1+v^2)\,\text{Sin}[u],\ v\}$.

1) The maximum Gauss curvature is **9.****

2) The maximum Gauss curvature is **2.****

3) The maximum Gauss curvature is **6.****

4) The maximum Gauss curvature is **0.****

5) The maximum Gauss curvature is **4.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$

$r(t) = \{ (t+4)\sin(2t)\,(6\cos(17t)+10),\ (7t+2)\sin(t) \}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 696.611 2) 995.111 3) 1791.11 4) 597.111

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-4)\,(x-2)\,(x-1)\,x & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2 and the moment t= 0.001 by means of a Fourier series of order 8 .

1) u(2 , 0.001) = ***.9***

2) u(2 , 0.001) = ***.1***

3) u(2 , 0.001) = ***.3***

4) u(2 , 0.001) = ***.4***

5) u(2 , 0.001) = ***.0***

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 23

Exercise 1

Consider the domain $D_1 \equiv 9x^2 - 6xy + 9y^2 = 1$ and the point $q = (-7, 5)$. Compute the diameter or largest distance between they two, diam(D_1, q), and the points of D_1 where it is attained.

1) The point of D_1 furthest from q is (* , ***.*1**)

2) The point of D_1 furthest from q is (* , ***.*2**)

3) The point of D_1 furthest from q is (* , ***.*0**)

4) The point of D_1 furthest from q is (* , ***.*3**)

5) The point of D_1 furthest from q is (* , ***.*9**)

Exercise 2

Compute $\int_D (y^2 + y^3)\, dx\, dy\, dz$ for $D = \{2 \le x^8 y^7 z^4 \le 4,\ 5x^9 z^8 \le y^4 \le 13x^9 z^8,\ 9y^8 \le x^7 z \le 13y^8,\ x > 0,\ y > 0,\ z > 0\}$

1) 1.70065

2) -0.699351

3) 1.10065

4) 0.000649331

5) -0.999351

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-v + (1 + v^2)\,\text{Cos}[u],\ 2v + (1 + v^2)\,\text{Sin}[u],\ -3v - 2(1 + v^2)\,\text{Sin}[u]\}$.

1) The maximum Gauss curvature is **1.****

2) The maximum Gauss curvature is **4.****

3) The maximum Gauss curvature is **7.****

4) The maximum Gauss curvature is **0.****

5) The maximum Gauss curvature is **9.****

Exercise 4

Consider the vector field $F(x,y,z) = \{-9xy + 6xz + \cos[y^2],\ -4 + \sin[2x^2],\ e^{-2x^2-y^2} + z\}$ and the surface

$$S \equiv \left(\frac{6+x}{4}\right)^2 + \left(\frac{-8+y}{5}\right)^2 + \left(\frac{8+z}{1}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −43 867.3   2) 25 922.7   3) −9969.32   4) −30 906.3

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0,\ \mathrm{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -2x & 0 \le x \le 1 \\ \frac{2x}{3} - \frac{8}{3} & 1 \le x \le 4 \end{cases} & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 1 , t= 0.007 , by separation of variables by means of a Fourier series of order 8.

1) u(1 , 0.007) = **7.****

2) u(1 , 0.007) = **9.****

3) u(1 , 0.007) = **8.****

4) u(1 , 0.007) = **0.****

5) u(1 , 0.007) = **1.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 24

Exercise 1

Consider the domain $D_1 \equiv 9x^2 + 6xy + 7y^2 = 1$ and the point $q = (6, -1)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \bigcup \{q\}$ is **4.****

2) The diameter of $D_1 \bigcup \{q\}$ is **6.****

3) The diameter of $D_1 \bigcup \{q\}$ is **5.****

4) The diameter of $D_1 \bigcup \{q\}$ is **0.****

5) The diameter of $D_1 \bigcup \{q\}$ is **3.****

Exercise 2

Compute $\int_D (2y + z)\, dx\, dy\, dz$ for $D = \{8z \leq x^4 y^8 \leq 15z,\ 3y^9 \leq z^8 \leq 10y^9,\ 3x^2 z^2 \leq y^3 \leq 12x^2 z^2,\ x > 0,\ y > 0,\ z > 0\}$

1) -1.19165

2) 1.70835

3) 0.208347

4) 0.608347

5) 0.408347

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{u,\ 26 + 10u + u^2 + 3v + v^2,\ 26 + 12u + u^2 + 2v + v^2\}$.

1) The maximum Gauss curvature is **9.****

2) The maximum Gauss curvature is **4.****

3) The maximum Gauss curvature is **3.****

4) The maximum Gauss curvature is **5.****

5) The maximum Gauss curvature is **6.****

Exercise 4

Consider the vector field $F(x,y,z) =$ $\{-3 + e^{y^2-2z^2} - 6xz,\ -x - 7z + \text{Sin}[x^2],\ -2xy - 2xz + \text{Sin}[2x^2 + 2y^2]\}$ and the surface

$$S \equiv \left(\frac{-3+x}{5}\right)^2 + \left(\frac{7+y}{9}\right)^2 + \left(\frac{-8+z}{4}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 48 860.2   2) 77 361.4   3) −154 720.   4) −40 715.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t->\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 5x & 0 \le x \le 1 \\ 2x+3 & 1 \le x \le 3 \\ -\frac{9x}{\pi-3} + \frac{27}{\pi-3} + 9 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.002 , by separation of variables by means of a Fourier series of order 9.

1) u(2 , 0.002) = **9.****

2) u(2 , 0.002) = **7.****

3) u(2 , 0.002) = **2.****

4) u(2 , 0.002) = **1.****

5) u(2 , 0.002) = **8.****

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Exercise 1

Consider the domain $D_1 \equiv 4x^2 - 8xy + 9y^2 = 1$ and the point $q = (9, 0)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \bigcup \{q\}$ is ***.5***

2) The diameter of $D_1 \bigcup \{q\}$ is ***.3***

3) The diameter of $D_1 \bigcup \{q\}$ is ***.6***

4) The diameter of $D_1 \bigcup \{q\}$ is ***.4***

5) The diameter of $D_1 \bigcup \{q\}$ is ***.7***

Exercise 2

Compute $\int_D (3\,x\,z)\,dx\,dy\,dz$ for $D = \{7x^4 \le y^9 z^9 \le 14x^4,\ 9 \le y^2 z^7 \le 10,\ 5y^9 \le x z^9 \le 12y^9,\ x > 0,\ y > 0,\ z > 0\}$

1) 1.90257

2) 0.00257165

3) -1.39743

4) 1.90257

5) -1.39743

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{5\,\text{Cos}[v] + 4\,\text{Cos}[u]\,\text{Sin}[v],\ 20\,\text{Cos}[v] + 5\,\text{Sin}[u]\,\text{Sin}[v],\ 5\,\text{Cos}[v]\}$.

1) The maximum Gauss curvature is **2.****

2) The maximum Gauss curvature is **8.****

3) The maximum Gauss curvature is **4.****

4) The maximum Gauss curvature is **0.****

5) The maximum Gauss curvature is **3.****

Exercise 4

Consider the vector field $F(x,y,z) = \{4x - 9xyz + \text{Sin}[2y^2], -7 + \text{Cos}[x^2 - z^2], 4 + 4z + \text{Sin}[x^2 + y^2]\}$ and the surface

$$S \equiv \left(\frac{x}{3}\right)^2 + \left(\frac{3+y}{4}\right)^2 + \left(\frac{-2+z}{6}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −44 874.4   2) −7478.44   3) 54 225.   4) 18 698.8

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(2,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-2)^2\,(x-1)\,x & 0 \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{19}{10}$ and the moment t= 0.008 by means of a Fourier series of order 12 .

1) $u\left(\frac{19}{10}, 0.008\right)$ = ***.3***

2) $u\left(\frac{19}{10}, 0.008\right)$ = ***.4***

3) $u\left(\frac{19}{10}, 0.008\right)$ = ***.7***

4) $u\left(\frac{19}{10}, 0.008\right)$ = ***.6***

5) $u\left(\frac{19}{10}, 0.008\right)$ = ***.1***

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 26

Exercise 1

Consider the domains $D_1 \equiv 125 - 60x + 8x^2 - 50y + 12xy + 5y^2 \leq 1$ and $D_2 \equiv 281 - 64x + 4x^2 - 10y + y^2 \leq 100$. Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (**0.**** , *)

2) The point of D_1 closest to D_2 is (**2.**** , *)

3) The point of D_1 closest to D_2 is (**1.**** , *)

4) The point of D_1 closest to D_2 is (**5.**** , *)

5) The point of D_1 closest to D_2 is (**7.**** , *)

Exercise 2

Compute the volume of the domain limited by the plane $6x + 5z == 2$ and the paraboloid $z == 2x^2 + 9y^2$ and the semiplanes $-9x - 9y \geq 0$ and $-6x + 3y \geq 0$.

1) 0.285924

2) 0.103418

3) 0.33964

4) 0.0941028

5) 0.095886

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{(1 + 2v^2)\,\text{Cos}[u],\ 2(1 + 2v^2)\,\text{Cos}[u] + (1 + 2v^2)\,\text{Sin}[u],\ v\}$.

1) The maximum Gauss curvature is **0.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **5.****

4) The maximum Gauss curvature is **4.****

5) The maximum Gauss curvature is **3.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{e^{-2y^2+z^2} - 9yz,\ 9x - 6xy + \mathrm{Sin}[2x^2 + z^2],\ 6xyz + \mathrm{Cos}[x^2]\right\}$ and the surface

$$S \equiv \left(\frac{-5+x}{6}\right)^2 + \left(\frac{1+y}{3}\right)^2 + \left(\frac{-3+z}{2}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) −39 811. 2) −9047.79 3) 11 762.6 4) −1809.39

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 2x & 0 \le x \le 2 \\ 8-2x & 2 \le x \le 4 \end{cases} & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1 and the moment t= 0.006 by means of a Fourier series of order 10 .

1) u(1 , 0.006) = **6.****

2) u(1 , 0.006) = **1.****

3) u(1 , 0.006) = **2.****

4) u(1 , 0.006) = **9.****

5) u(1 , 0.006) = **4.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 27

Exercise 1

Given the system

$2 - 2ux^2 - 2x^3 - 2uy - 2xy + 2y^2 + 2uy^2 - 3y^3 == -13$
$2ux - 2u^2x - 2x^2 - 3ux^2 - 3u^2y + 2xy + uxy - 2uy^2 == 16$

determine if it is possible to solve for variables x , y in terms of variable u arround the point p=(x, y, u)=(3, -1, -2). Compute if possible the approximate values of (x, y) for (u)=(-1.9) by means of the tangent affine function at p.

1) (x, y) ≃ (2.97174, -1.08696)

2) (x, y) ≃ (2.67174, -1.38696)

3) (x, y) ≃ (3.27174, -1.58696)

4) (x, y) ≃ (3.37174, -0.986957)

5) (x, y) ≃ (3.47174, -0.686957)

Exercise 2

Compute the volume of the domain limited by the plane $x + 6z == 5$ and the paraboloid $z == 8x^2 + 4y^2$ and the semiplanes $8x + 5y \geq 0$ and $5x - 3y \geq 0$.

1) 0.0487816

2) 0.161356

3) 0.172945

4) 0.0571304

5) 0.106555

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = $\{-4v + (1 + 3v^2)\,\text{Cos}[u] - 12(1 + 3v^2)\,\text{Sin}[u],\ 2v + 5(1 + 3v^2)\,\text{Sin}[u],\ v + 2(1 + 3v^2)\,\text{Sin}[u]\}$.

1) The maximum Gauss curvature is **4.****

2) The maximum Gauss curvature is **3.****

3) The maximum Gauss curvature is **8.****

4) The maximum Gauss curvature is **1.****

5) The maximum Gauss curvature is **0.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{-3\,x\,y + \text{Sin}[z^2],\ -3\,x + 8\,z - \text{Sin}[2\,x^2 + z^2],\ 7 + e^{y^2} - 2\,y\,z\right\}$ and the surface

$S \equiv \left(\frac{7+x}{9}\right)^2 + \left(\frac{-8+y}{2}\right)^2 + \left(\frac{6+z}{3}\right)^2 = 1$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −4523.79 2) −9047.79 3) −10 857.4 4) −39 811.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = 2\,(x-1)^2\left(x - \frac{3}{10}\right)x^2 & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = \frac{3}{10}$, $t = 0.008$, by separation of variables by means of a Fourier series of order 8.

1) $u\left(\frac{3}{10}, 0.008\right)$ = ***.***4

2) $u\left(\frac{3}{10}, 0.008\right)$ = ***.***1

3) $u\left(\frac{3}{10}, 0.008\right)$ = ***.***7

4) $u\left(\frac{3}{10}, 0.008\right)$ = ***.***0

5) $u\left(\frac{3}{10}, 0.008\right)$ = ***.***6

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 28

Exercise 1

Given the system

$v x - y + 3 u v y - x y - u x y + 3 x^2 y == 10$
$-2 u x - 2 v y == -20$

determine if it is possible to solve for variables x , y in terms of variables u , v arround the point p=(x , y , u , v)=(-2 , 0 , -5 , -5). Compute if possible the approximate values of (x , y) for (u , v)=(-4.7 , -5.1) by means of the tangent affine function at p.

1) (x , y) ≃ (-2.51036 , 0.490361)

2) (x , y) ≃ (-2.51036 , -0.509639)

3) (x , y) ≃ (-2.61036 , 0.190361)

4) (x , y) ≃ (-2.61036 , -0.509639)

5) (x , y) ≃ (-2.11036 , -0.00963855)

Exercise 2

Compute the volume of the domain limited by the plane $4 x + 10 z == 8$ and the paraboloid $z == 8 x^2 + 5 y^2$ and the semiplanes $-7 x - 9 y \geq 0$ and $-2 x + 5 y \geq 0$.

1) 0.0927724

2) 0.0361307

3) 0.125893

4) 0.0952545

5) 0.0266403

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-100 - 11 u - u^2 - 15 v - v^2, v, 100 + 12 u + u^2 + 18 v + v^2\}$.

1) The maximum Gauss curvature is **4.****

2) The maximum Gauss curvature is **2.****

3) The maximum Gauss curvature is **1.****

4) The maximum Gauss curvature is **6.****

5) The maximum Gauss curvature is **9.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{9 + z - \mathrm{Sin}\left[y^2 - 2 z^2\right],\ 8 + e^{x^2+2 z^2} + 2 y,\ e^{2 x^2-y^2} - 4 x\right\}$ and the surface

$$S \equiv \left(\frac{-7 + x}{1}\right)^2 + \left(\frac{-1 + y}{3}\right)^2 + \left(\frac{7 + z}{9}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 226.195   2) -519.605   3) 135.795   4) 678.195

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 3\,(x-3)\,(x-2)\,x^2\,(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.002$ by means of a Fourier series of order 10 .

1) u(1 , 0.002) = *1*.****

2) u(1 , 0.002) = *0*.****

3) u(1 , 0.002) = *6*.****

4) u(1 , 0.002) = *9*.****

5) u(1 , 0.002) = *5*.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 29

Exercise 1

Given the system

$-2\,w\,x_2\,x_4 == 4$

$2\,u\,x_2 + w\,x_2 - 2\,x_1\,x_4^2 == -8$

$w^2\,x_1 - u^2\,x_2 + 3\,v^2\,x_2 - u\,x_1\,x_4 == -93$

$u\,x_1\,x_3 - 2\,v\,x_3\,x_4 == -40$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v, w arround the point $p = (x_1, x_2, x_3, x_4, u, v, w) = (3, -2, -5, 1, 0, -4, 1)$. Compute if possible $\frac{\partial x_1}{\partial w}(0, -4, 1)$.

1) $\frac{\partial x_1}{\partial w}(0, -4, 1) = \frac{73}{13}$

2) $\frac{\partial x_1}{\partial w}(0, -4, 1) = \frac{512}{91}$

3) $\frac{\partial x_1}{\partial w}(0, -4, 1) = \frac{513}{91}$

4) $\frac{\partial x_1}{\partial w}(0, -4, 1) = \frac{510}{91}$

5) $\frac{\partial x_1}{\partial w}(0, -4, 1) = \frac{514}{91}$

Exercise 2

Compute $\int_D (x\,y)\,dx\,dy$ for $D = \{2\,y^3 \le x^4 \le 6\,y^3,\ 6\,x^9 \le y^7 \le 11\,x^9,\ x > 0,\ y > 0\}$

1) 1.34896×10^{37}

2) -6.74481×10^{35}

3) 6.74481×10^{36}

4) 1.01172×10^{37}

5) 1.956×10^{37}

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{2v - 7(1+v^2)\,\mathrm{Cos}[u],\ (1+v^2)\,\mathrm{Sin}[u],\ v - 4(1+v^2)\,\mathrm{Cos}[u]\}$.

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **4.****

3) The maximum Gauss curvature is **2.****

4) The maximum Gauss curvature is **1.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \dashrightarrow R^2$
$r(t) = \{(7t+9)\sin(2t)(\cos(6t)+1),\ (4t+6)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 294.699 2) 324.099 3) 30.0985 4) 265.299

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = 2(x-4)(x-3)x & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=1$ and the moment $t=0.004$ by means of a Fourier series of order 11 .

1) u(1 , 0.004) = *0*.****

2) u(1 , 0.004) = *8*.****

3) u(1 , 0.004) = *2*.****

4) u(1 , 0.004) = *1*.****

5) u(1 , 0.004) = *6*.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 30

Exercise 1

Consider the domains $D_1 \equiv 936 - 162\,x + 9\,x^2 - 82\,y + 4\,x\,y + 3\,y^2 \leq 1$ and $D_2 \equiv 720 - 120\,x + 8\,x^2 + 120\,y - 4\,x\,y + 8\,y^2 \leq 1000$.
Compute the distance between they two, $d(D_1, D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (* , **3.****)

2) The point of D_1 closest to D_2 is (* , **8.****)

3) The point of D_1 closest to D_2 is (* , **7.****)

4) The point of D_1 closest to D_2 is (* , **9.****)

5) The point of D_1 closest to D_2 is (* , **4.****)

Exercise 2

Compute the volume of the domain limited by the plane $4\,x + z == 7$ and the paraboloid $z == 8\,x^2 + 9\,y^2$ and the semiplanes $9\,x - 7\,y \geq 0$ and $-6\,y \geq 0$.

1) 11.9164

2) 10.3504

3) 10.9253

4) 2.64879

5) 12.5554

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-\text{Cos}[u]\,\text{Sin}[v] + 6\,\text{Sin}[u]\,\text{Sin}[v],\ -\text{Cos}[u]\,\text{Sin}[v] + 3\,\text{Sin}[u]\,\text{Sin}[v],\ 3\,\text{Cos}[v]\}$.

1) The maximum Gauss curvature is **1.****

2) The maximum Gauss curvature is **3.****

3) The maximum Gauss curvature is **5.****

4) The maximum Gauss curvature is **7.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Consider the vector field $F(x,y,z) = \{8xz - 8xyz + \text{Sin}[2z^2],\ e^{2z^2} - 6x - 4yz,\ -\text{Sin}[2x^2]\}$ and the surface

$$S \equiv \left(\frac{-2+x}{4}\right)^2 + \left(\frac{-3+y}{4}\right)^2 + \left(\frac{-7+z}{2}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 24 396. 2) 56 298.2 3) −18 765.8 4) 52 545.

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = -\left((x-1)\left(x-\frac{3}{5}\right)x\right) & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{4}{5}$

and the moment $t = 0.009$ by means of a Fourier series of order 12 .

1) $u\left(\frac{4}{5}, 0.009\right)$ = ***.*1**

2) $u\left(\frac{4}{5}, 0.009\right)$ = ***.*9**

3) $u\left(\frac{4}{5}, 0.009\right)$ = ***.*8**

4) $u\left(\frac{4}{5}, 0.009\right)$ = ***.*4**

5) $u\left(\frac{4}{5}, 0.009\right)$ = ***.*7**

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Exercise 1

Consider the domain $D_1 \equiv 7x^2 + 2xy + y^2 = 1$ and the point $q = (-5, 3)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is ***.**5*

2) The distance between D_1 and q is ***.**8*

3) The distance between D_1 and q is ***.**2*

4) The distance between D_1 and q is ***.**0*

5) The distance between D_1 and q is ***.**6*

Exercise 2

Compute $\int_D (3x)\,dx\,dy\,dz$ for D=
$\{3x^2 z^8 \le 1 \le 11 x^2 z^8,\ 6 x z^4 \le y \le 15 x z^4,\ 9 y^7 z^3 \le x^2 \le 15 y^7 z^3,\ x > 0,\ y > 0,\ z > 0\}$

1) 2184.12

2) 1680.09

3) 5040.28

4) 168.009

5) 504.028

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {4 Cos[u] Sin[v] + 3 Sin[u] Sin[v], 4 Cos[u] Sin[v] + 6 Sin[u] Sin[v], 3 Cos[v] − 4 Cos[u] Sin[v] + 6 Sin[u] Sin[v]}.

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **4.****

3) The maximum Gauss curvature is **3.****

4) The maximum Gauss curvature is **5.****

5) The maximum Gauss curvature is **2.****

Exercise 4

Consider the vector field $F(x,y,z) = \{2y - 4xz + \mathrm{Sin}[2y^2 - z^2],\ e^{-2x^2+2z^2} + 3x,\ z + \mathrm{Sin}[2x^2 + 2y^2]\}$ and the surface

$$S \equiv \left(\frac{-5+x}{1}\right)^2 + \left(\frac{-6+y}{5}\right)^2 + \left(\frac{-5+z}{5}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −8158.68 2) −1193.68 3) −1591.68 4) −1989.68

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 3(x-3)(x-1)x(x-\pi)^2 & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = -3(x-2)(x-1)x(x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 2 and the moment t= 0.009 by means of a Fourier series of order 12 .

1) u(2 , 0.009) = **4.****

2) u(2 , 0.009) = **3.****

3) u(2 , 0.009) = **9.****

4) u(2 , 0.009) = **0.****

5) u(2 , 0.009) = **7.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 32

Exercise 1

Given the function

$f(x, y, z) = -8 + 6x - x^2 - 6y + y^2 + 2z - z^2$ defined over the domain D≡ $\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {-5.20818, ?, 0.189748} and {3, 3, 1} is not a local maximum of f.

2) We have a minimum at {?, 0.547883, -0.110252} and {3, 3, 1} is not a local maximum of f.

3) We have a minimum at {?, 3, 1} and {3, 3, 1} is not a local minimum of f.

4) We have a minimum at {-4.70818, 0.847883, ?} and {3, 3, 1} is not a local maximum of f.

5) We have a minimum at {?, 0.0478827, 0.0897483} and {3, 3, 1} is not a saddle point of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $4x + 2z == 6$ and the paraboloid $z == 8x^2 + 6y^2$ and the semiplanes $5x - 5y \geq 0$ and $5x - 8y \geq 0$.

1) -0.117528

2) -6.36816

3) -0.599119

4) -0.69539

5) -1.39687

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {-4 Cos[v] + 4 Cos[u] Sin[v] + 10 Sin[u] Sin[v], 2 Sin[u] Sin[v], 4 Cos[v] - 4 Sin[u] Sin[v]}.

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **8.****

3) The maximum Gauss curvature is **9.****

4) The maximum Gauss curvature is **3.****

5) The maximum Gauss curvature is **5.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{-y\,z + \mathrm{Sin}[2\,z^2],\ 3\,x - \mathrm{Sin}[2\,x^2 - 2\,z^2],\ -4 + e^{2\,x^2-y^2}\right\}$ and the surface

$$S \equiv \left(\frac{-8+x}{7}\right)^2 + \left(\frac{-8+y}{9}\right)^2 + \left(\frac{-2+z}{8}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 0.   2) 0.5   3) 1.5   4) 3.6

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0,\ \mathrm{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = -3\,(x-1)\left(x-\frac{9}{10}\right)\left(x-\frac{3}{5}\right)x^2 & 0 \le x \le 1 \\ 0 & \mathrm{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=\frac{1}{5}$, $t=0.009$, by separation of variables by means of a Fourier series of order 10.

1) $u\left(\frac{1}{5}, 0.009\right)$ = ***.*0**

2) $u\left(\frac{1}{5}, 0.009\right)$ = ***.*1**

3) $u\left(\frac{1}{5}, 0.009\right)$ = ***.*2**

4) $u\left(\frac{1}{5}, 0.009\right)$ = ***.*5**

5) $u\left(\frac{1}{5}, 0.009\right)$ = ***.*8**

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 33

Exercise 1

Given the function

$f(x, y, z) = -20 - 2x + x^2 + 6y - y^2 + 4z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {1, ?, 2} and {1, 3, 2} is not a saddle point of f.

2) We have a maximum at {-3.25279, ?, 0.234907} and {1, 3, 2} is not a local maximum of f.

3) We have a maximum at {-3.25279, ?, 0.990809} and {1, 3, 2} is not a saddle point of f.

4) We have a maximum at {-2.62287, 1.25984, ?} and {1, 3, 2} is not a local maximum of f.

5) We have a maximum at {-2.24492, ?, 0.864825} and {1, 3, 2} is not a saddle point of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $6x + 2z == 2$ and the paraboloid $z == 6x^2 + 6y^2$.

1) 2.34538

2) 0.219962

3) 0.389252

4) 1.98132

5) 0.494964

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {-4 Cos[v] + 2 Cos[u] Sin[v] + 3 Sin[u] Sin[v], 8 Cos[v] - 4 Cos[u] Sin[v] - 3 Sin[u] Sin[v], -6 Cos[v] + 4 Cos[u] Sin[v] + 3 Sin[u] Sin[v]} .

1) The maximum Gauss curvature is **4.****

2) The maximum Gauss curvature is **1.****

3) The maximum Gauss curvature is **9.****

4) The maximum Gauss curvature is **7.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]\longrightarrow R^2$
$r(t)=\{(5t+8)\sin(2t)(6\cos(5t)+9),\ (9t+6)\sin(t)(6\cos(5t)+9)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 76171.4   2) 46549.5   3) 42317.8   4) 67708.

Exercise 5

$$\begin{cases} (1+2t+t^2)\frac{\partial u}{\partial t}(x,t)=9(2+2t)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -\frac{4x}{3} & 0\le x\le 3 \\ \frac{4x}{\pi-3}-\frac{12}{\pi-3}-4 & 3\le x\le \pi \end{cases} & 0\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=2$, $t=0.005$, by separation of variables by means of a Fourier series of order 12.

1) u(2 , 0.005) = **8.****

2) u(2 , 0.005) = **5.****

3) u(2 , 0.005) = **6.****

4) u(2 , 0.005) = **7.****

5) u(2 , 0.005) = **2.****

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Exercise 1

Given the function

$f(x, y, z) = 7 - 2x + x^2 - 2y + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{9} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {?, 1.4, 0.1} and {1, 1, 0} is not a saddle point of f.

2) We have a minimum at {1.4, ?, -0.1} and {1, 1, 0} is not a local maximum of f.

3) We have a minimum at {1, 1, ?} and {0, -1, -2} is not a local maximum of f.

4) We have a minimum at {0.5, 1.1, ?} and {1, 1, 0} is not a local maximum of f.

5) We have a minimum at {?, 1.5, 0.2} and {1, 1, 0} is not a saddle point of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $6x + 5z == 8$ and the paraboloid $z == x^2 + 2y^2$ and the semiplanes $x - 7y \geq 0$ and $-8x + 6y \geq 0$.

1) 0.733471

2) 1.80692

3) 0.535683

4) 1.13301

5) 2.54203

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{v + (1 + v^2)\,\text{Cos}[u],\ 3v + (1 + v^2)\,\text{Cos}[u] + (1 + v^2)\,\text{Sin}[u],\ v\}$.

1) The maximum Gauss curvature is **0.****

2) The maximum Gauss curvature is **2.****

3) The maximum Gauss curvature is **5.****

4) The maximum Gauss curvature is **6.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Consider the vector field $F(x,y,z) = \{8y + \text{Cos}[z^2], -1 - \text{Sin}[x^2], 5 + 4z + \text{Sin}[x^2 + y^2]\}$ and the surface

$$S \equiv \left(\frac{-1 + x}{7}\right)^2 + \left(\frac{2 + y}{8}\right)^2 + \left(\frac{9 + z}{8}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 7506.31   2) −3002.09   3) 3002.71   4) 19515.9

Exercise 5

$$\begin{cases} (1+t)\frac{\partial u}{\partial t}(x,t) = 9(1)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 6x & 0 \le x \le 1 \\ \frac{15}{2} - \frac{3x}{2} & 1 \le x \le 3 \\ -\frac{3x}{\pi-3} + \frac{9}{\pi-3} + 3 & 3 \le x \le \pi \\ 0 & \text{True} \end{cases} & 0 \le x \le \pi \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = 2$, $t = 0.002$, by separation of variables by means of a Fourier series of order 11.

1) u(2 , 0.002) = **6.****

2) u(2 , 0.002) = **4.****

3) u(2 , 0.002) = **2.****

4) u(2 , 0.002) = **0.****

5) u(2 , 0.002) = **5.****

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Exercise 1

Given the function

$f(x, y, z) = -2 + x^2 + 2y - y^2 - 6z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {0., 0.0908771, ?} and {0, 1, 3} is not a local maximum of f.

2) We have a maximum at {0, 1, ?} and {0, 1, 3} is not a local minimum of f.

3) We have a maximum at {-0.1, ?, -4.59484} and {0, 1, 3} is not a local maximum of f.

4) We have a maximum at {-0.2, ?, -5.39484} and {0, 1, 3} is not a saddle point of f.

5) We have a maximum at {0.5, -0.309123, ?} and {0, 1, 3} is not a saddle point of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $5x + 8z == 1$ and the paraboloid $z == x^2 + y^2$.

1) 0.0947468

2) 0.0723088

3) 0.0778735

4) 0.253217

5) 0.0787645

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {Cos[u] (4 + 2 Cos[v]), (4 + 2 Cos[v]) Sin[u] + 2 Sin[v], Cos[u] (4 + 2 Cos[v]) + Sin[v]}.

1) The maximum Gauss curvature is **1.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **9.****

4) The maximum Gauss curvature is **4.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \longrightarrow R^2$

$r(t) = \{(4t+2)\sin(2t)(4\cos(15t)+10),\ (t+1)\sin(t)(4\cos(15t)+10)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 4810.54 2) 5131.24 3) 962.139 4) 3207.04

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9(8 - t)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-1)x^2(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = 2$, $t = 0.008$, by separation of variables by means of a Fourier series of order 11.

1) u(2 , 0.008) = **6.****

2) u(2 , 0.008) = **4.****

3) u(2 , 0.008) = **9.****

4) u(2 , 0.008) = **1.****

5) u(2 , 0.008) = **8.****

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Exercise 1

Given the system

$-2\,y\,z^2 - 2\,u_2^3 + 2\,x\,y\,u_4 == -100$

$3\,x\,u_4 - z^2\,u_4 == -15$

$-2\,y\,z\,u_1 - 2\,u_1^2\,u_2 + 3\,u_1\,u_2^2 == 100$

determine if it is possible to solve for variables x, y, z in terms of variables u_1, u_2, u_3, u_4 arround the point $p=(x, y, z, u_1, u_2, u_3, u_4)=(4, 2, 3, -5, -2, -4, -5)$. Compute if possible $\frac{\partial z}{\partial u_4}(-5, -2, -4, -5)$.

1) $\frac{\partial z}{\partial u_4}(-5, -2, -4, -5) = \frac{12}{19}$

2) $\frac{\partial z}{\partial u_4}(-5, -2, -4, -5) = \frac{13}{19}$

3) $\frac{\partial z}{\partial u_4}(-5, -2, -4, -5) = \frac{9}{19}$

4) $\frac{\partial z}{\partial u_4}(-5, -2, -4, -5) = \frac{10}{19}$

5) $\frac{\partial z}{\partial u_4}(-5, -2, -4, -5) = \frac{11}{19}$

Exercise 2

Compute $\int_D (x^2 + y)\,dx\,dy$ for $D=\{3 \le x^7\,y^{16} \le 12,\ 7 \le x^3\,y^7 \le 16,\ x > 0,\ y > 0\}$

1) -1.72574

2) 1.47426

3) 0.0742586

4) 0.374259

5) 0.974259

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-23u + 4u^2 + 4(45 - 12v + v^2), -180 + 24u - 4u^2 + 49v - 4v^2, 2(45 - 6u + u^2 - 12v + v^2)\}$.

1) The maximum Gauss curvature is **5.****

2) The maximum Gauss curvature is **2.****

3) The maximum Gauss curvature is **3.****

4) The maximum Gauss curvature is **1.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r : [0, \pi] \longrightarrow R^2$
$r(t) = \{(8t + 3)\sin(2t)(6\cos(20t) + 9), (9t + 6)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 4732.13 2) 3154.93 3) 3943.53 4) 5126.43

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = -3(x-4)(x-2)x & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 3 and the moment t= 0.006 by means of a Fourier series of order 10 .

1) u(3 , 0.006) = **9.****

2) u(3 , 0.006) = **8.****

3) u(3 , 0.006) = **4.****

4) u(3 , 0.006) = **7.****

5) u(3 , 0.006) = **2.****

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Exercise 1

Given the function

$f(x, y, z) = 7 + 4x - x^2 - 2y + y^2 - 4z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {-3.1289, 0.139336, ?} and {2, 1, 2} is not a local maximum of f.

2) We have a minimum at {-2.88045, ?, 1.24227} and {2, 1, 2} is not a local minimum of f.

3) We have a minimum at {-2.38354, ?, 1.61495} and {2, 1, 2} is not a local minimum of f.

4) We have a minimum at {2, ?, 2} and {2, 1, 2} is not a local minimum of f.

5) We have a minimum at {?, 0.0151095, 0.869586} and {2, 1, 2} is not a local maximum of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $7x + 7z == 8$ and the paraboloid $z == x^2 + y^2$.

1) 5.62524

2) 3.04743

3) 8.05252

4) 1.02863

5) 0.2692

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {-Cos[v] + 5 Cos[u] Sin[v], 3 Sin[u] Sin[v], 4 Cos[v] - 15 Cos[u] Sin[v]}.

1) The maximum Gauss curvature is **8.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **9.****

4) The maximum Gauss curvature is **0.****

5) The maximum Gauss curvature is **4.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \longrightarrow R^2$

$r(t)=\{(9t+2)\sin(2t)(3\cos(16t)+9),\ (5t+7)\sin(t)(3\cos(16t)+9)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 28 587.3 2) 8576.39 3) 5717.69 4) 17 152.5

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t)=u(4,t)=0 & 0\le t \\ u(x,0)=\begin{cases} \frac{x}{2} & 0\le x\le 2 \\ 2-\frac{x}{2} & 2\le x\le 4 \end{cases} & 0\le x\le 4 \\ \frac{\partial}{\partial t}u(x,0)=3(x-4)^2(x-2)x & 0.\le x\le 4 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1
and the moment t= 0.006 by means of a Fourier series of order 11 .

1) u (1 , 0.006) = ***.2***

2) u (1 , 0.006) = ***.3***

3) u (1 , 0.006) = ***.1***

4) u (1 , 0.006) = ***.4***

5) u (1 , 0.006) = ***.6***

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Exercise 1

Given the system

$-3 u^2 v + 3 u x^2 + 2 v x^2 + 2 x^3 + 2 v y^2 == -96$
$u^3 - x^3 - 2 v x y - 2 y^2 + 3 y^3 == 15$

determine if it is possible to solve for variables x , y in terms of variables u , v arround the point p = (x , y , u , v) = (-2 , 3 , -2 , -4). Compute if possible $\frac{\partial x}{\partial v}(-2, -4)$.

1) $\frac{\partial x}{\partial v}(-2, -4) = -\frac{657}{2408}$

2) $\frac{\partial x}{\partial v}(-2, -4) = -\frac{47}{172}$

3) $\frac{\partial x}{\partial v}(-2, -4) = -\frac{655}{2408}$

4) $\frac{\partial x}{\partial v}(-2, -4) = -\frac{659}{2408}$

5) $\frac{\partial x}{\partial v}(-2, -4) = -\frac{82}{301}$

Exercise 2

Compute $\int_D (x y)\, dx dy$ for $D = \{5 x^9 y^5 \le 1 \le 9 x^9 y^5,\ 7 x^{25} y^{14} \le 1 \le 13 x^{25} y^{14},\ x > 0,\ y > 0\}$

1) -9.63734×10^{10}

2) 9.63734×10^{11}

3) 2.02384×10^{12}

4) -6.74614×10^{11}

5) 2.50571×10^{12}

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = $\{-v + (1 + 2 v^2) \mathrm{Cos}[u],\ 5 v - 2 (1 + 2 v^2) \mathrm{Cos}[u] + (1 + 2 v^2) \mathrm{Sin}[u],\ 3 v - 2 (1 + 2 v^2) \mathrm{Cos}[u]\}$.

1) The maximum Gauss curvature is **3.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **0.****

4) The maximum Gauss curvature is **7.****

5) The maximum Gauss curvature is **9.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \longrightarrow R^2$
$r(t)=\{(3t+8)\sin(2t)(9\cos(7t)+10),\ (6t+1)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 1774.44 2) 197.636 3) 3548.34 4) 1971.54

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(\pi,t)=0 & 0\le t \\ u(x,0)=3(x-3)x^2(x-\pi) & 0\le x\le\pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1 and the moment t= 0.003 by means of a Fourier series of order 12 .

1) u(1 , 0.003) = *9*.****

2) u(1 , 0.003) = *7*.****

3) u(1 , 0.003) = *6*.****

4) u(1 , 0.003) = *1*.****

5) u(1 , 0.003) = *3*.****

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Exercise 1

Given the function

$f(x, y, z) = -6x + x^2 + 2y - y^2 - 2z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, 1, 1} and {3, 1, 1} is not a local maximum of f.

2) We have a maximum at {?, 0.180753, -0.283092} and {3, 1, 1} is not a local minimum of f.

3) We have a maximum at {?, -0.319247, 0.116908} and {3, 1, 1} is not a local maximum of f.

4) We have a maximum at {-3.25739, 0.0807529, ?} and {3, 1, 1} is not a local maximum of f.

5) We have a maximum at {-3.45739, ?, -0.0830924} and {3, 1, 1} is not a local minimum of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $5x + 4z == 6$ and the paraboloid $z == 9x^2 + 9y^2$.

1) 0.893258

2) 0.487933

3) 0.13175

4) 0.210334

5) 0.415754

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {3 Cos[u] (2 + Cos[v]) + (2 + Cos[v]) Sin[u] + 2 Sin[v], -2 Cos[u] (2 + Cos[v]) + (2 + Cos[v]) Sin[u], 2 Cos[u] (2 + Cos[v]) + Sin[v]} .

1) The maximum Gauss curvature is **9.****

2) The maximum Gauss curvature is **8.****

3) The maximum Gauss curvature is **0.****

4) The maximum Gauss curvature is **4.****

5) The maximum Gauss curvature is **6.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \longrightarrow R^2$

$r(t)=\{(t+8)\sin(2t)(4\cos(8t)+7), (2t+2)\sin(t)(4\cos(8t)+7)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 5302.4 2) 4923.7 3) 6817.2 4) 3787.6

Exercise 5

$$\begin{cases} (1+6t+3t^2)\frac{\partial u}{\partial t}(x,t)=9(6+6t)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -15x & 0\le x\le \frac{1}{5} \\ 50x-13 & \frac{1}{5}\le x\le \frac{2}{5} \\ \frac{35}{3}-\frac{35x}{3} & \frac{2}{5}\le x\le 1 \end{cases} & 0\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=\frac{2}{5}$, $t=0.001$, by separation of variables by means of a Fourier series of order 11.

1) $u(\frac{2}{5}, 0.001)$ = **7.****

2) $u(\frac{2}{5}, 0.001)$ = **2.****

3) $u(\frac{2}{5}, 0.001)$ = **9.****

4) $u(\frac{2}{5}, 0.001)$ = **5.****

5) $u(\frac{2}{5}, 0.001)$ = **8.****

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Exercise 1

Consider the domain $D_1 \equiv 6x^2 + 6xy + 3y^2 = 1$ and the point $q = (-8, -2)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \bigcup \{q\}$ is ***.*9**

2) The diameter of $D_1 \bigcup \{q\}$ is ***.*4**

3) The diameter of $D_1 \bigcup \{q\}$ is ***.*0**

4) The diameter of $D_1 \bigcup \{q\}$ is ***.*7**

5) The diameter of $D_1 \bigcup \{q\}$ is ***.*1**

Exercise 2

Compute $\int_D (3x + z)\,dx\,dy\,dz$ for $D=$ $\{8 \le x^4 y^2 z^8 \le 14,\ 3x^6 z^7 \le y^3 \le 5x^6 z^7,\ 2xy^4 \le z^3 \le 11xy^4,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.109205

2) 0.709205

3) 0.209205

4) 0.00920477

5) −1.2908

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{u,\ 2(113 + 16u + u^2 + 14v + v^2),\ -452 - 63u - 4u^2 - 55v - 4v^2\}$.

1) The maximum Gauss curvature is **7.****

2) The maximum Gauss curvature is **0.****

3) The maximum Gauss curvature is **1.****

4) The maximum Gauss curvature is **2.****

5) The maximum Gauss curvature is **9.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{e^{-2y^2} - 3y,\ 6xyz + \text{Cos}[2z^2],\ -5x + \text{Cos}[2y^2]\right\}$ and the surface

$$S \equiv \left(\frac{-5+x}{1}\right)^2 + \left(\frac{7+y}{5}\right)^2 + \left(\frac{3+z}{9}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) −16 964.6 2) −8482.1 3) 42 412.9 4) −28 840.1

Exercise 5

$$\begin{cases} (1+9t+2t^2)\frac{\partial u}{\partial t}(x,t) = 9(9+4t)\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 3x & 0 \le x \le 1 \\ \frac{9}{2} - \frac{3x}{2} & 1 \le x \le 3 \end{cases} & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 1 , t= 0.005 , by separation of variables by means of a Fourier series of order 8.

1) u(1 , 0.005) = **1.****

2) u(1 , 0.005) = **3.****

3) u(1 , 0.005) = **8.****

4) u(1 , 0.005) = **9.****

5) u(1 , 0.005) = **2.****

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Exercise 1

Consider the domain $D_1 \equiv 6x^2 + 10xy + 5y^2 = 1$ and the point $q = (-1, -4)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The diameter of $D_1 \bigcup \{q\}$ is ***.3***

2) The diameter of $D_1 \bigcup \{q\}$ is ***.2***

3) The diameter of $D_1 \bigcup \{q\}$ is ***.4***

4) The diameter of $D_1 \bigcup \{q\}$ is ***.8***

5) The diameter of $D_1 \bigcup \{q\}$ is ***.0***

Exercise 2

Compute $\int_D (y^2 + z^2)\, dx dy dz$ for $D = \{3 \leq x^8 y^4 z^2 \leq 6,\ 9 \leq x^8 y^3 z^4 \leq 18,\ 9 z^6 \leq y^7 \leq 11 z^6,\ x > 0,\ y > 0,\ z > 0\}$

1) 1.44292

2) −0.157077

3) 0.342923

4) 0.842923

5) 1.14292

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{-v + (1 + 2v^2)\,\text{Cos}[u] - 2(1 + 2v^2)\,\text{Sin}[u],\ (1 + 2v^2)\,\text{Cos}[u] - (1 + 2v^2)\,\text{Sin}[u],\ -v - 2(1 + 2v^2)\,\text{Sin}[u]\}$.

1) The maximum Gauss curvature is **4.****

2) The maximum Gauss curvature is **7.****

3) The maximum Gauss curvature is **0.****

4) The maximum Gauss curvature is **3.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Consider the vector field $F(x,y,z) =$
$\left\{e^{-2 z^2} - 4 x y z,\ 6 + 4 x y + \mathrm{Cos}\left[2 x^2 - z^2\right],\ 7 y - 5 y z - \mathrm{Sin}\left[2 y^2\right]\right\}$ and the surface

$$S \equiv \left(\frac{7 + x}{4}\right)^2 + \left(\frac{7 + y}{8}\right)^2 + \left(\frac{9 + z}{7}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −160 917. 2) −229 881. 3) −505 738. 4) 229 881.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0,\ \mathrm{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -6x & 0 \le x \le 1 \\ 15x - 21 & 1 \le x \le 2 \\ 27 - 9x & 2 \le x \le 3 \end{cases} & 0 \le x \le 3 \\ 0 & \mathrm{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = 2$
, $t = 0.009$, by separation of variables by means of a Fourier series of order 9.

1) u(2 , 0.009) = **6.****

2) u(2 , 0.009) = **0.****

3) u(2 , 0.009) = **7.****

4) u(2 , 0.009) = **5.****

5) u(2 , 0.009) = **4.****

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Exercise 1

Given the system

$-x^3 - 3\,y\,u_5 == -10$
$-x^2\,y == 3$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (1, -3, 0, 0, -5, 3, -1)$. Compute if possible $\frac{\partial x}{\partial u_5}(0, 0, -5, 3, -1)$.

1) $\frac{\partial x}{\partial u_5}(0, 0, -5, 3, -1) = -\frac{2}{5}$

2) $\frac{\partial x}{\partial u_5}(0, 0, -5, 3, -1) = 0$

3) $\frac{\partial x}{\partial u_5}(0, 0, -5, 3, -1) = \frac{1}{5}$

4) $\frac{\partial x}{\partial u_5}(0, 0, -5, 3, -1) = -\frac{3}{5}$

5) $\frac{\partial x}{\partial u_5}(0, 0, -5, 3, -1) = -\frac{1}{5}$

Exercise 2

Compute $\int_D (x + x^2)\,dx\,dy$ for $D = \{6\,y^3 \le x \le 12\,y^3,\ 7\,y^2 \le x \le 13\,y^2,\ x > 0,\ y > 0\}$

1) 13654.1

2) 7186.36

3) 10779.5

4) −7186.36

5) −2155.91

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) =
{3 Cos[u] (3 + 2 Cos[v]) + 4 (3 + 2 Cos[v]) Sin[u] + 2 Sin[v],
2 Cos[u] (3 + 2 Cos[v]) + 3 (3 + 2 Cos[v]) Sin[u] + Sin[v], Sin[v]} .

1) The maximum Gauss curvature is **7.****

2) The maximum Gauss curvature is **9.****

3) The maximum Gauss curvature is **8.****

4) The maximum Gauss curvature is **3.****

5) The maximum Gauss curvature is **5.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]\longrightarrow R^2$
$r(t)=\{(4t+6)\sin(2t)(6\cos(14t)+10),\ (9t+6)\sin(t)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 6071.66 2) 2698.66 3) 337.563 4) 3373.26

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(2,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -6x & 0\le x\le 1 \\ 6x-12 & 1\le x\le 2 \end{cases} & 0\le x\le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=\frac{6}{5}$
and the moment t = 0.006 by means of a Fourier series of order 12 .

1) $u(\frac{6}{5}, 0.006)$ = **5.****

2) $u(\frac{6}{5}, 0.006)$ = **6.****

3) $u(\frac{6}{5}, 0.006)$ = **2.****

4) $u(\frac{6}{5}, 0.006)$ = **8.****

5) $u(\frac{6}{5}, 0.006)$ = **4.****

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Exercise 1

Given the system

$v x + 2 v z + 2 y z^2 == -88$

$-v x^2 + 3 z == 41$

$-3 u + x - v z == 11$

determine if it is possible to solve for variables x , y , z in terms of variables u , v arround the point p = (x , y , z , u , v) = (5 , -5 , -3 , -4 , -2) . Compute if possible $\frac{\partial y}{\partial u}(-4, -2)$.

1) $\frac{\partial y}{\partial u}(-4, -2) = -\frac{563}{111}$

2) $\frac{\partial y}{\partial u}(-4, -2) = -\frac{560}{111}$

3) $\frac{\partial y}{\partial u}(-4, -2) = -\frac{559}{111}$

4) $\frac{\partial y}{\partial u}(-4, -2) = -\frac{187}{37}$

5) $\frac{\partial y}{\partial u}(-4, -2) = -\frac{562}{111}$

Exercise 2

Compute $\int_D (x^2)\, dx dy$ for $D = \{8 \le x^5 y^4 \le 10,\ 4 \le x y \le 9,\ x > 0,\ y > 0\}$

1) 2.00016

2) 0.00015641

3) 0.500156

4) 1.20016

5) 1.40016

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) =
$\{-340-53u-4u^2-49v-4v^2, -340-54u-4u^2-47v-4v^2, 850+133u+10u^2+122v+10v^2\}$.

1) The maximum Gauss curvature is **8.****

2) The maximum Gauss curvature is **3.****

3) The maximum Gauss curvature is **7.****

4) The maximum Gauss curvature is **2.****

5) The maximum Gauss curvature is **5.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \dashrightarrow R^2$
$r(t) = \{(6t+8)\sin(2t)(3\cos(11t)+7), (t+5)\sin(t)\}$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 1964.56 2) 1200.86 3) 764.455 4) 1091.76

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t)=u(4,t)=0 & 0 \le t \\ u(x,0)=-2(x-4)(x-3)(x-1)x^2 & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 3 and the moment t= 0.003 by means of a Fourier series of order 10 .

1) u(3 , 0.003) = ***.*6**

2) u(3 , 0.003) = ***.*4**

3) u(3 , 0.003) = ***.*7**

4) u(3 , 0.003) = ***.*5**

5) u(3 , 0.003) = ***.*0**

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Exercise 1

Given the system

$2u - v x^2 - u v y == -70$

$-2 + 2u - u^3 - 3x^2 - 2 v y + 2 u x y - 3 v y^2 == -246$

determine if it is possible to solve for variables x , y in terms of variables u , v arround the point p = (x , y , u , v) = (5 , -1 , 5 , 4). Compute if possible $\frac{\partial x}{\partial u}(5,4)$.

1) $\frac{\partial x}{\partial u}(5,4) = -\frac{15}{43}$

2) $\frac{\partial x}{\partial u}(5,4) = -\frac{77}{215}$

3) $\frac{\partial x}{\partial u}(5,4) = -\frac{76}{215}$

4) $\frac{\partial x}{\partial u}(5,4) = -\frac{79}{215}$

5) $\frac{\partial x}{\partial u}(5,4) = -\frac{78}{215}$

Exercise 2

Compute $\int_D (3y^3)\, dx dy$ for $D = \{8x^2 \le y^3 \le 15x^2,\ 4y^5 \le x^3 \le 11y^5,\ x > 0,\ y > 0\}$

1) -1.3

2) -1.9

3) -2.

4) -1.2

5) 1.69853×10^{-24}

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {4 Cos[u] Sin[v] - 10 Sin[u] Sin[v], 5 Sin[u] Sin[v], 5 Cos[v] + 4 Cos[u] Sin[v]} .

1) The maximum Gauss curvature is **5.****

2) The maximum Gauss curvature is **7.****

3) The maximum Gauss curvature is **8.****

4) The maximum Gauss curvature is **1.****

5) The maximum Gauss curvature is **6.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r : [0,\pi] \longrightarrow R^2$

$r(t) = \{(4t + 8)\sin(2t)\,(7\cos(14t) + 8),\ (3t + 3)\sin(t)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 356.3 2) 237.8 3) 948.8 4) 1185.8

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = -3\,(x-4)^2\,(x-2)\,x & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$
and the moment $t = 0.007$ by means of a Fourier series of order 11 .

1) u(2 , 0.007) = ***.7***

2) u(2 , 0.007) = ***.6***

3) u(2 , 0.007) = ***.5***

4) u(2 , 0.007) = ***.1***

5) u(2 , 0.007) = ***.8***

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Exercise 1

Given the system

$-2 u v + 2 v w + 3 u v x + w x + 2 w^2 y + 2 u y^2 == 46$
$-u v x + 3 x^2 y == 4$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p = (x , y , u , v , w) = (-2 , 1 , 4 , -1 , 3). Compute if possible the approximate values of (x , y) for (u , v , w) = (3.9 , -1.2 , 2.8) by means of the tangent affine function at p.

1) (x , y) ≃ (-2.18049 , 0.296341)

2) (x , y) ≃ (-2.38049 , 1.19634)

3) (x , y) ≃ (-2.48049 , 0.796341)

4) (x , y) ≃ (-2.68049 , 0.696341)

5) (x , y) ≃ (-2.18049 , 0.696341)

Exercise 2

Compute the volume of the domain limited by the plane $2 x + 7 z == 4$ and the paraboloid $z == 3 x^2 + 4 y^2$ and the semiplanes $5 x + 4 y \geq 0$ and $9 x - 7 y \geq 0$.

1) 0.0359294

2) 0.141403

3) 0.0376885

4) 0.0513667

5) 0.0814193

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {Cos[u] (4 + 2 Cos[v]) + 2 (4 + 2 Cos[v]) Sin[u], -3 Cos[u] (4 + 2 Cos[v]) - 5 (4 + 2 Cos[v]) Sin[u], Cos[u] (4 + 2 Cos[v]) + 2 (4 + 2 Cos[v]) Sin[u] + Sin[v]} .

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **8.****

3) The maximum Gauss curvature is **1.****

4) The maximum Gauss curvature is **5.****

5) The maximum Gauss curvature is **4.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{e^{-2z^2} + 3xy,\ -3xz - \mathrm{Sin}\left[2x^2\right],\ -3x - \mathrm{Sin}\left[2x^2 - y^2\right]\right\}$ and the surface

$$S \equiv \left(\frac{-2+x}{6}\right)^2 + \left(\frac{-2+y}{4}\right)^2 + \left(\frac{-8+z}{8}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 4825.49 2) 22 678. 3) −14 474.5 4) −12 544.5

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0,\ \mathrm{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -7x & 0 \le x \le 1 \\ x-8 & 1 \le x \le 2 \\ 6x-18 & 2 \le x \le 3 \end{cases} & 0 \le x \le 3 \\ 0 & \mathrm{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x=1$, $t=0.005$, by separation of variables by means of a Fourier series of order 11.

1) u(1 , 0.005) = **6.****

2) u(1 , 0.005) = **8.****

3) u(1 , 0.005) = **7.****

4) u(1 , 0.005) = **5.****

5) u(1 , 0.005) = **9.****

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Exercise 1

Given the system

$-x\,u_1 + 3\,u_3^2 - 3\,u_4^2 + y\,u_4^2 + 3\,u_3\,u_4^2 == -313$

$2\,x\,u_1^2 + 3\,y\,u_4 - 3\,u_3\,u_4 == 9$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (-3, 2, 4, 4, -5, 5)$. Compute if possible the approximate values of (x, y) for $(u_1, u_2, u_3, u_4) = (4.2, 4.3, -5.3, 5.3)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (-3.79709, 3.86047)$

2) $(x, y) \simeq (-4.59709, 4.06047)$

3) $(x, y) \simeq (-3.89709, 4.56047)$

4) $(x, y) \simeq (-3.89709, 4.36047)$

5) $(x, y) \simeq (-4.09709, 4.26047)$

Exercise 2

Compute the volume of the domain limited by the plane $7x + 3z == 5$ and the paraboloid $z == 9x^2 + 2y^2$ and the semiplanes $-9x + 7y \geq 0$ and $-2x + 4y \geq 0$.

1) 0.654871

2) 1.58311

3) 0.189517

4) 1.37478

5) 0.736083

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{\mathrm{Cos}[u]\,(4 + 3\,\mathrm{Cos}[v]) + 2\,(4 + 3\,\mathrm{Cos}[v])\,\mathrm{Sin}[u],\ (4 + 3\,\mathrm{Cos}[v])\,\mathrm{Sin}[u],\ -\mathrm{Cos}[u]\,(4 + 3\,\mathrm{Cos}[v]) + 3\,(4 + 3\,\mathrm{Cos}[v])\,\mathrm{Sin}[u] + \mathrm{Sin}[v]\}$.

1) The maximum Gauss curvature is **8.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **1.****

4) The maximum Gauss curvature is **7.****

5) The maximum Gauss curvature is **5.****

Exercise 4

Consider the vector field $F(x,y,z) = \{x z + 5 y z + \text{Cos}[z^2],\ 4 - \text{Sin}[2 x^2 - 2 z^2],\ e^{-2 x^2} + 6 z\}$ and the surface

$$S \equiv \left(\frac{-8 + x}{2}\right)^2 + \left(\frac{-9 + y}{5}\right)^2 + \left(\frac{z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 2412.31   2) −2110.19   3) 1005.31   4) −1708.19

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4 \frac{\partial^2 u}{\partial x^2}(x,t) & 0 < x < \pi,\ 0 < t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = 3\,(x - 3)\,x\,(x - \pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.008$ by means of a Fourier series of order 12 .

1) u(1 , 0.008) = *0*.****

2) u(1 , 0.008) = *8*.****

3) u(1 , 0.008) = *1*.****

4) u(1 , 0.008) = *4*.****

5) u(1 , 0.008) = *3*.****

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Exercise 1

Consider the domain $D_1 \equiv 8x^2 + 12xy + 8y^2 = 1$ and the point $q = (9, -8)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is ***.**7*

2) The distance between D_1 and q is ***.**9*

3) The distance between D_1 and q is ***.**3*

4) The distance between D_1 and q is ***.**5*

5) The distance between D_1 and q is ***.**4*

Exercise 2

Compute $\int_D (3y^2)\,dx\,dy\,dz$ for $D=$
$\{5y^3 \leq x^8 z^8 \leq 13y^3,\ 2y \leq xz \leq 5y,\ 2x^5 y^9 \leq z^5 \leq 9x^5 y^9,\ x > 0,\ y > 0,\ z > 0\}$

1) -1.29796

2) 0.802043

3) 0.802043

4) -0.397957

5) 0.00204313

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) =$
$\{4\,\mathrm{Cos}[u]\,\mathrm{Sin}[v],\ -2\,\mathrm{Cos}[v] - 4\,\mathrm{Cos}[u]\,\mathrm{Sin}[v] + 2\,\mathrm{Sin}[u]\,\mathrm{Sin}[v],\ \mathrm{Cos}[v]\}$.

1) The maximum Gauss curvature is **1.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **9.****

4) The maximum Gauss curvature is **4.****

5) The maximum Gauss curvature is **0.****

Exercise 4

Consider the vector field $F(x,y,z) =$
$\{e^{y^2-2z^2} - 9xy, -6yz + 3xyz - \mathrm{Sin}[x^2], e^{-y^2} - 3x - 7xyz\}$ and the surface

$$S \equiv \left(\frac{7+x}{4}\right)^2 + \left(\frac{-4+y}{9}\right)^2 + \left(\frac{7+z}{6}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 757842. 2) −599957. 3) 315768. 4) 663111.

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \mathrm{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = (x-3)(x-2)x^2(x-\pi) & 0 \le x \le \pi \\ 0 & \mathrm{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 1 , t= 0.002 , by separation of variables by means of a Fourier series of order 11.

1) u(1 , 0.002) = **5.****

2) u(1 , 0.002) = **1.****

3) u(1 , 0.002) = **2.****

4) u(1 , 0.002) = **4.****

5) u(1 , 0.002) = **7.****

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Exercise 1

Consider the domain $D_1 \equiv 5x^2 + 4xy + 6y^2 = 1$ and the point $q = (7, -1)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 furthest from q is (* , **0.****)

2) The point of D_1 furthest from q is (* , **1.****)

3) The point of D_1 furthest from q is (* , **2.****)

4) The point of D_1 furthest from q is (* , **3.****)

5) The point of D_1 furthest from q is (* , **5.****)

Exercise 2

Compute $\int_D (z + z^3)\, dx\, dy\, dz$ for $D = \{6xy^3 \le z^2 \le 14xy^3,\ 2x^9z^5 \le y^8 \le 3x^9z^5,\ x \le y^7z^6 \le 5x,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.104312

2) 1.10431

3) 0.00431236

4) −0.195688

5) 1.60431

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{u + v,\ 5v - 4(80 + 16u + u^2 + 8v + v^2),\ 80 + 16u + u^2 + 7v + v^2\}$.

1) The maximum Gauss curvature is **7.****

2) The maximum Gauss curvature is **5.****

3) The maximum Gauss curvature is **6.****

4) The maximum Gauss curvature is **3.****

5) The maximum Gauss curvature is **9.****

Exercise 4

Consider the vector field $F(x,y,z) =$
$\{-9 + e^{-y^2+z^2} + 8\,x\,y\,z,\ 8\,x\,y + \mathrm{Cos}[x^2],\ e^{x^2-2\,y^2} + 7\,x - 5\,x\,z\}$ and the surface

$$S \equiv \left(\frac{-2+x}{7}\right)^2 + \left(\frac{-8+y}{7}\right)^2 + \left(\frac{-5+z}{1}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 160 587.   2) 86 985.   3) 127 132.   4) 66 911.7

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -3\,(x-2)\,x\,(x-\pi)^2 & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} 4\,x & 0 \le x \le 2 \\ 16 - 4\,x & 2 \le x \le 3 \\ -\frac{4\,x}{\pi-3} + \frac{12}{\pi-3} + 4 & 3 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$
and the moment $t = 0.005$ by means of a Fourier series of order 8 .

1) u (1 , 0.005) = *1*.****

2) u (1 , 0.005) = *7*.****

3) u (1 , 0.005) = *9*.****

4) u (1 , 0.005) = *8*.****

5) u (1 , 0.005) = *0*.****

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Exercise 1

Consider the domain $D_1 \equiv 9x^2 - 2xy + 8y^2 = 1$ and the point $q = (6, -9)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 closest to q is (* , ***.2***)

2) The point of D_1 closest to q is (* , ***.7***)

3) The point of D_1 closest to q is (* , ***.5***)

4) The point of D_1 closest to q is (* , ***.8***)

5) The point of D_1 closest to q is (* , ***.1***)

Exercise 2

Compute $\int_D (x z^2)\, dx\, dy\, dz$ for $D=$
$\{3z^3 \le x^7 y^8 \le 7z^3,\ 8x^9 y^7 z^4 \le 1 \le 10 x^9 y^7 z^4,\ x y^8 z \le 1 \le 5 x y^8 z,\ x>0,\ y>0,\ z>0\}$

1) 0.400148

2) 0.200148

3) -1.09985

4) 0.000148494

5) 1.00015

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) =$
{-4 Cos[v] + Cos[u] Sin[v], 8 Cos[v] - Cos[u] Sin[v] + 3 Sin[u] Sin[v], 2 Cos[v]}.

1) The maximum Gauss curvature is **8.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **5.****

4) The maximum Gauss curvature is **2.****

5) The maximum Gauss curvature is **0.****

Exercise 4

Consider the vector field $F(x,y,z) = \left\{-4\,y - \text{Sin}\left[y^2 + 2\,z^2\right],\ \text{Sin}\left[x^2\right],\ e^{-x^2} - 7\,x\,z\right\}$ and the surface

$$S \equiv \left(\frac{8+x}{1}\right)^2 + \left(\frac{7+y}{8}\right)^2 + \left(\frac{-4+z}{5}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 4691.89 2) 0.890059 3) −20 639.5 4) 9382.89

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-2)\,x^2\,(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 1 , t= 0.002 , by separation of variables by means of a Fourier series of order 8.

1) u(1 , 0.002) = **5.****

2) u(1 , 0.002) = **0.****

3) u(1 , 0.002) = **7.****

4) u(1 , 0.002) = **4.****

5) u(1 , 0.002) = **2.****

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Exercise 1

Given the system

$-2\,v\,x_2 + 2\,u\,x_3 == -30$
$u\,x_1^2 - x_2 + x_4 == 95$
$-2\,u^2\,x_1 - 3\,x_1^2 + 3\,x_2\,x_3\,x_4 == 121$
$v\,x_1\,x_2 == -15$

determine if it is possible to solve for variables x_1, x_2, x_3, x_4 in terms of variables u, v arround the point $p = (x_1, x_2, x_3, x_4, u, v) = (-5, 1, -3, -4, 4, 3)$. Compute if possible $\frac{\partial x_3}{\partial u}(4,3)$.

1) $\frac{\partial x_3}{\partial u}(4,3) = \frac{7}{8}$

2) $\frac{\partial x_3}{\partial u}(4,3) = \frac{787}{896}$

3) $\frac{\partial x_3}{\partial u}(4,3) = \frac{783}{896}$

4) $\frac{\partial x_3}{\partial u}(4,3) = \frac{393}{448}$

5) $\frac{\partial x_3}{\partial u}(4,3) = \frac{785}{896}$

Exercise 2

Compute $\int_D (x\,y^2)\,dx\,dy$ for $D = \{4\,y^7 \le x^2 \le 12\,y^7,\ 7\,x \le y^4 \le 9\,x,\ x > 0,\ y > 0\}$

1) 4.07905×10^{28}

2) 9.78972×10^{28}

3) -2.85534×10^{28}

4) 1.14213×10^{29}

5) 2.85534×10^{28}

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{2v + (1+3v^2)\,\mathrm{Cos}[u],\ -2v - (1+3v^2)\,\mathrm{Cos}[u] + (1+3v^2)\,\mathrm{Sin}[u],\ v - 2(1+3v^2)\,\mathrm{Sin}[u]\}$.

1) The maximum Gauss curvature is **3.****

2) The maximum Gauss curvature is **6.****

3) The maximum Gauss curvature is **4.****

4) The maximum Gauss curvature is **7.****

5) The maximum Gauss curvature is **1.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \dashrightarrow R^2$
$r(t) = \{(3t+4)\sin(2t)\,(8\cos(17t)+8),\ (t+3)\sin(t)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 436.617 2) 611.017 3) 698.217 4) 523.817

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(2,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 8x & 0 \le x \le 1 \\ 16 - 8x & 1 \le x \le 2 \end{cases} & 0 \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{13}{10}$
and the moment t = 0.003 by means of a Fourier series of order 8.

1) $u\left(\frac{13}{10}, 0.003\right)$ = **5.****

2) $u\left(\frac{13}{10}, 0.003\right)$ = **3.****

3) $u\left(\frac{13}{10}, 0.003\right)$ = **6.****

4) $u\left(\frac{13}{10}, 0.003\right)$ = **4.****

5) $u\left(\frac{13}{10}, 0.003\right)$ = **9.****

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Exercise 1

Consider the domains $D_1 \equiv 448 + 80\,x + 9\,x^2 + 32\,y - 8\,x\,y + 6\,y^2 \leq 1$ and $D_2 \equiv 232 - 8\,x + x^2 - 72\,y + 6\,y^2 \leq 1000$. Compute the distance between they two, $d(D_1,D_2)$, and the points where it is attained.

1) The point of D_1 closest to D_2 is (* , ***.1***)

2) The point of D_1 closest to D_2 is (* , ***.5***)

3) The point of D_1 closest to D_2 is (* , ***.7***)

4) The point of D_1 closest to D_2 is (* , ***.0***)

5) The point of D_1 closest to D_2 is (* , ***.6***)

Exercise 2

Compute the volume of the domain limited by the plane $x + 10\,z == 4$ and the paraboloid $z == x^2 + 5\,y^2$ and the semiplanes $-9\,x - 2\,y \geq 0$ and $-5\,x + y \geq 0$.

1) 0.0869194

2) 0.129707

3) 0.230027

4) 0.0610914

5) 0.127782

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {2 Cos[v] + 5 Cos[u] Sin[v] - 8 Sin[u] Sin[v], 2 Sin[u] Sin[v], Cos[v] + 4 Sin[u] Sin[v]} .

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **8.****

3) The maximum Gauss curvature is **1.****

4) The maximum Gauss curvature is **3.****

5) The maximum Gauss curvature is **9.****

Exercise 4

Consider the vector field $F(x,y,z) = \{9 - 5y + \mathrm{Sin}[2z^2],\ 9z - \mathrm{Sin}[x^2 - 2z^2],\ e^{2x^2 - y^2} - 17xz\}$ and the surface

$$S \equiv \left(\frac{2+x}{1}\right)^2 + \left(\frac{y}{8}\right)^2 + \left(\frac{3+z}{1}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 4556.35 2) 1139.35 3) −2391.55 4) 1253.25

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t) = u(2,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 9x & 0 \le x \le 1 \\ 18 - 9x & 1 \le x \le 2 \end{cases} & 0 \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{9}{10}$ and the moment $t = 0.006$ by means of a Fourier series of order 8 .

1) $u\left(\frac{9}{10}, 0.006\right)$ = **1.****

2) $u\left(\frac{9}{10}, 0.006\right)$ = **7.****

3) $u\left(\frac{9}{10}, 0.006\right)$ = **5.****

4) $u\left(\frac{9}{10}, 0.006\right)$ = **8.****

5) $u\left(\frac{9}{10}, 0.006\right)$ = **9.****

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Exercise 1

Given the system

$-u v^2 + 3 u x - x^2 - 3 y - x y - u x y == -11$
$3 u^3 - u^2 v - 3 v^3 - 3 u v x - 3 u y - 3 u y^2 == 40$

determine if it is possible to solve for variables x, y in terms of variables u, v arround the point $p=(x, y, u, v)=(2, 3, -2, 2)$. Compute if possible the approximate values of (x, y) for $(u, v)=(-2.2, 2.2)$ by means of the tangent affine function at p.

1) $(x, y) \simeq (2.04043, 3.41702)$

2) $(x, y) \simeq (2.04043, 3.11702)$

3) $(x, y) \simeq (2.04043, 2.61702)$

4) $(x, y) \simeq (2.34043, 3.01702)$

5) $(x, y) \simeq (2.64043, 3.11702)$

Exercise 2

Compute the volume of the domain limited by the plane $3 x + 2 z == 4$ and the paraboloid $z == 9 x^2 + 7 y^2$ and the semiplanes $5 x - 9 y \geq 0$ and $2 x - 5 y \geq 0$.

1) -0.485125

2) -0.294421

3) -0.0071004

4) -0.277687

5) -1.18496

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{(1 + v^2) \text{Cos}[u], (1 + v^2) \text{Sin}[u], v - (1 + v^2) \text{Sin}[u]\}$.

1) The maximum Gauss curvature is **1.****

2) The maximum Gauss curvature is **5.****

3) The maximum Gauss curvature is **4.****

4) The maximum Gauss curvature is **2.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Consider the vector field $F(x,y,z) = \{\mathrm{Cos}[2y^2 - 2z^2], -3 + 7yz + \mathrm{Sin}[2x^2 + 2z^2], -8 - 7xyz + \mathrm{Cos}[x^2 + y^2]\}$ and the surface

$$S \equiv \left(\frac{-7+x}{1}\right)^2 + \left(\frac{-7+y}{7}\right)^2 + \left(\frac{2+z}{3}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −31 403.4   2) 53 387.4   3) −78 509.4   4) −69 088.2

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-3)\,x^2\,(x-\pi)^2 & 0 \le x \le \pi \\ 0 & \mathrm{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.01$ by means of a Fourier series of order 8 .

1) u(1 , 0.01) = *3*.****

2) u(1 , 0.01) = *8*.****

3) u(1 , 0.01) = *7*.****

4) u(1 , 0.01) = *4*.****

5) u(1 , 0.01) = *1*.****

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 53

Exercise 1

Consider the domain $D_1 \equiv 4x^2 + 6xy + 9y^2 = 1$ and the point $q = (9, -7)$.
Compute the distance between they two, $d(D_1, q)$, and the points of D_1 where it is attained.

1) The distance between D_1 and q is ***.*9**

2) The distance between D_1 and q is ***.*1**

3) The distance between D_1 and q is ***.*3**

4) The distance between D_1 and q is ***.*4**

5) The distance between D_1 and q is ***.*0**

Exercise 2

Compute $\int_D (4xz)\,dx\,dy\,dz$ for $D =$
$\{2z^4 \le x^7 y^8 \le 6z^4,\ 2y^2 \le xz^3 \le 5y^2,\ 9x^4 y^4 \le z^8 \le 15x^4 y^4,\ x > 0,\ y > 0,\ z > 0\}$

1) −1.52342

2) 0.0765756

3) 1.47658

4) −0.0234244

5) 0.776576

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) =$
$\{(1 + 3v^2)\,\mathrm{Cos}[u],\ (1 + 3v^2)\,\mathrm{Sin}[u],\ v + 2(1 + 3v^2)\,\mathrm{Sin}[u]\}$.

1) The maximum Gauss curvature is **4.****

2) The maximum Gauss curvature is **2.****

3) The maximum Gauss curvature is **6.****

4) The maximum Gauss curvature is **0.****

5) The maximum Gauss curvature is **5.****

Exercise 4

Consider the vector field $F(x,y,z) = \{-4\,x\,z + \text{Cos}[2\,y^2],\ 3\,y + \text{Cos}[2\,z^2],\ -6 - 7\,x\,z + \text{Cos}[y^2]\}$ and the surface

$$S \equiv \left(\frac{8+x}{4}\right)^2 + \left(\frac{7+y}{4}\right)^2 + \left(\frac{-1+z}{3}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) −12 163.4 2) 11 058.4 3) 30 962.8 4) −9951.79

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-2)\,x\,(x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.002 , by separation of variables by means of a Fourier series of order 10.

1) u(2 , 0.002) = ***.*5**

2) u(2 , 0.002) = ***.*6**

3) u(2 , 0.002) = ***.*2**

4) u(2 , 0.002) = ***.*0**

5) u(2 , 0.002) = ***.*3**

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Exercise 1

Given the function

$f(x, y, z) = -16 + 6x - x^2 - 2y + y^2 + 6z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {-2.72475, 0.458332, ?} and {3, 1, 3} is not a local maximum of f.

2) We have a minimum at {3, 1, ?} and {3, 1, 3} is not a saddle point of f.

3) We have a minimum at {-3.22475, 0.958332, ?} and {3, 1, 3} is not a local maximum of f.

4) We have a minimum at {?, 0.858332, -1.10487} and {3, 1, 3} is not a saddle point of f.

5) We have a minimum at {?, -0.0416679, -0.504873} and {3, 1, 3} is not a saddle point of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $7x + 4z == 8$ and the paraboloid $z == 6x^2 + 6y^2$.

1) 1.18509

2) 0.610747

3) 5.27432

4) 1.51204

5) 2.83348

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = $\{(1 + v^2)\,\text{Cos}[u],\ (1 + v^2)\,\text{Cos}[u] + (1 + v^2)\,\text{Sin}[u],\ v + 2\,(1 + v^2)\,\text{Cos}[u] + 3\,(1 + v^2)\,\text{Sin}[u]\}$.

1) The maximum Gauss curvature is **9.****

2) The maximum Gauss curvature is **3.****

3) The maximum Gauss curvature is **1.****

4) The maximum Gauss curvature is **5.****

5) The maximum Gauss curvature is **7.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]\longrightarrow R^2$
$r(t)=\{(9t+8)\sin(2t)(\cos(20t)+7),\ (4t+5)\sin(t)(\cos(20t)+7)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 11 947.8   2) 29 015.8   3) 10 241.   4) 17 068.2

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=\begin{cases} \frac{10x}{3} & 0\le x\le \frac{3}{10} \\ \frac{14}{5}-6x & \frac{3}{10}\le x\le \frac{4}{5} \\ 10x-10 & \frac{4}{5}\le x\le 1 \end{cases} & 0\le x\le 1 \\ \frac{\partial}{\partial t}u(x,0)=-2(x-1)\left(x-\frac{4}{5}\right)\left(x-\frac{3}{5}\right)x & 0.\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=\frac{3}{5}$

and the moment t= 0.006 by means of a Fourier series of order 8 .

1) $u(\frac{3}{5}, 0.006) =$ ***.1***

2) $u(\frac{3}{5}, 0.006) =$ ***.4***

3) $u(\frac{3}{5}, 0.006) =$ ***.6***

4) $u(\frac{3}{5}, 0.006) =$ ***.7***

5) $u(\frac{3}{5}, 0.006) =$ ***.2***

Further Mathematics - Degree in Engineering - 2025/2026 Final Training Exam - January Call - Computers for serial number: 55

Exercise 1

Consider the domain $D_1 \equiv 4x^2 + 6xy + 7y^2 = 1$ and the point $q = (-3, 5)$. Compute the diameter or largest distance between they two, diam(D_1, q), and the points of D_1 where it is attained.

1) The point of D_1 furthest from q is (***.*4** , *)

2) The point of D_1 furthest from q is (***.*6** , *)

3) The point of D_1 furthest from q is (***.*1** , *)

4) The point of D_1 furthest from q is (***.*5** , *)

5) The point of D_1 furthest from q is (***.*2** , *)

Exercise 2

Compute $\int_D (3x^2 y)\,dx\,dy\,dz$ for $D = \{1 \le x^9 y^3 z^7 \le 7,\ 4x^3 \le y^5 z^5 \le 9x^3,\ 4x^5 z^8 \le y^7 \le 10x^5 z^8,\ x > 0,\ y > 0,\ z > 0\}$

1) −1.99018

2) −1.69018

3) 0.00982454

4) 0.909825

5) 0.509825

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {−2 Cos[v] − 12 Cos[u] Sin[v] − 2 Sin[u] Sin[v], 2 Cos[v] + 16 Cos[u] Sin[v] + 2 Sin[u] Sin[v], 3 Cos[v] + 20 Cos[u] Sin[v] + 2 Sin[u] Sin[v]} .

1) The maximum Gauss curvature is **7.****

2) The maximum Gauss curvature is **3.****

3) The maximum Gauss curvature is **4.****

4) The maximum Gauss curvature is **1.****

5) The maximum Gauss curvature is **5.****

Exercise 4

Consider the vector field $F(x,y,z) = \{5 x y + \text{Sin}[y^2 + 2 z^2], 3 z - 9 x z + \text{Cos}[2 x^2 + 2 z^2], -5 y - 7 z + \text{Cos}[x^2 - y^2]\}$ and the surface

$$S \equiv \left(\frac{2+x}{4}\right)^2 + \left(\frac{4+y}{3}\right)^2 + \left(\frac{2+z}{1}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −1357.17   2) −813.968   3) −2307.77   4) 3531.63

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0,\ \text{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -x & 0 \le x \le 1 \\ 5x - 6 & 1 \le x \le 2 \\ -\frac{4x}{\pi-2} + \frac{8}{\pi-2} + 4 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point x= 2 , t= 0.003 , by separation of variables by means of a Fourier series of order 9.

1) u(2 , 0.003) = **3.****

2) u(2 , 0.003) = **4.****

3) u(2 , 0.003) = **9.****

4) u(2 , 0.003) = **6.****

5) u(2 , 0.003) = **1.****

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Exercise 1

Given the system

$3 u y - x y == -8$
$3 - 3 u x + u w x - u y + 3 x y == 39$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p = (x , y , u , v , w) = (-5 , -4 , -1 , -5 , -1). Compute if possible the approximate values of (x , y) for (u , v , w) = (-0.7 , -5.1 , -0.9) by means of the tangent affine function at p.

1) (x , y) ≃ (-3.625 , -3.55)

2) (x , y) ≃ (-4.225 , -3.85)

3) (x , y) ≃ (-4.625 , -3.75)

4) (x , y) ≃ (-4.125 , -3.95)

5) (x , y) ≃ (-4.225 , -4.35)

Exercise 2

Compute the volume of the domain limited by the plane $4 x + 2 z == 9$ and the paraboloid $z == 9 x^2 + 4 y^2$ and the semiplanes $8 x - 9 y \geq 0$ and $-6 x + 4 y \geq 0$.

1) 0.295823

2) 1.35052

3) 0.767771

4) 0.190947

5) 0.511597

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {7 Cos[v] + 5 Cos[u] Sin[v] + 4 Sin[u] Sin[v], 2 Cos[v] + 2 Sin[u] Sin[v], Cos[v]} .

1) The maximum Gauss curvature is **8.****

2) The maximum Gauss curvature is **9.****

3) The maximum Gauss curvature is **5.****

4) The maximum Gauss curvature is **2.****

5) The maximum Gauss curvature is **0.****

Exercise 4

Consider the vector field $F(x,y,z) =$
$\left\{e^{-y^2} - 5\,x\,y + 5\,x\,y\,z,\ 3\,y\,z + \text{Cos}\left[x^2 - 2\,z^2\right],\ e^{-2\,x^2-y^2} + 9\,y\,z\right\}$ and the surface

$$S \equiv \left(\frac{4+x}{6}\right)^2 + \left(\frac{-3+y}{2}\right)^2 + \left(\frac{6+z}{2}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) −37 638.9   2) −14 476.5   3) −9650.97   4) 3860.43

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -x & 0 \le x \le 1 \\ 6\,x - 7 & 1 \le x \le 2 \\ -\frac{5\,x}{\pi-2} + \frac{10}{\pi-2} + 5 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = -2\,(x-3)\,x^2\,(x-\pi)^2 & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 2
and the moment t= 0.009 by means of a Fourier series of order 9 .

1) u(2 , 0.009) = **7.****

2) u(2 , 0.009) = **0.****

3) u(2 , 0.009) = **6.****

4) u(2 , 0.009) = **4.****

5) u(2 , 0.009) = **9.****

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Exercise 1

Consider the domain $D_1 \equiv x^2 - 2xy + 8y^2 = 1$ and the point $q = (-8, 3)$. Compute the diameter or largest distance between they two, $\text{diam}(D_1, q)$, and the points of D_1 where it is attained.

1) The point of D_1 furthest from q is (* , ***.*0**)

2) The point of D_1 furthest from q is (* , ***.*9**)

3) The point of D_1 furthest from q is (* , ***.*2**)

4) The point of D_1 furthest from q is (* , ***.*7**)

5) The point of D_1 furthest from q is (* , ***.*3**)

Exercise 2

Compute $\int_D (y^2)\, dx\, dy\, dz$ for $D = \{3y \le x^9 z^7 \le 12y,\ 7z^5 \le x^5 y^6 \le 9z^5,\ 2x^4 y^3 \le z^4 \le 6x^4 y^3,\ x > 0,\ y > 0,\ z > 0\}$

1) 1.63033

2) 1.33033

3) 0.730335

4) 2.03033

5) -0.669665

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {Cos[u] (3 + 2 Cos[v]) + 2 (3 + 2 Cos[v]) Sin[u] - Sin[v], -Cos[u] (3 + 2 Cos[v]) - (3 + 2 Cos[v]) Sin[u] + Sin[v], 4 (3 + 2 Cos[v]) Sin[u] + Sin[v]} .

1) The maximum Gauss curvature is **3.****

2) The maximum Gauss curvature is **1.****

3) The maximum Gauss curvature is **4.****

4) The maximum Gauss curvature is **9.****

5) The maximum Gauss curvature is **2.****

Exercise 4

Consider the vector field $F(x,y,z) = \{e^{-y^2-z^2} - 4x, -3xy + 9z - \mathrm{Sin}[x^2], e^{2y^2} + 4x\}$ and the surface

$$S \equiv \left(\frac{4+x}{6}\right)^2 + \left(\frac{-8+y}{4}\right)^2 + \left(\frac{9+z}{6}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 4342.99 2) 13028. 3) 7237.99 4) 4825.49

Exercise 5

$$\begin{cases} \frac{\partial^3 u}{\partial t^3}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0,\ \mathrm{Lim}_{t\to\infty} u(x,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 90x & 0 \le x \le \frac{1}{10} \\ 10 - 10x & \frac{1}{10} \le x \le 1 \end{cases} & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the value of the solution of this boundary problem at the point $x = \frac{7}{10}$, $t = 0.005$, by separation of variables by means of a Fourier series of order 12.

1) $u\left(\frac{7}{10}, 0.005\right)$ = **8.****

2) $u\left(\frac{7}{10}, 0.005\right)$ = **1.****

3) $u\left(\frac{7}{10}, 0.005\right)$ = **7.****

4) $u\left(\frac{7}{10}, 0.005\right)$ = **3.****

5) $u\left(\frac{7}{10}, 0.005\right)$ = **2.****

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Exercise 1

Given the function

$f(x, y, z) = 9 - 6x + x^2 + 4y - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a minimum at {3, ?, 0} and {3, 2, 0} is not a saddle point of f.

2) We have a minimum at {?, -3.65122, -0.3} and {3, 2, 0} is not a local maximum of f.

3) We have a minimum at {?, -3.35122, -0.4} and {3, 2, 0} is not a local minimum of f.

4) We have a minimum at {?, -3.85122, 0.} and {3, 2, 0} is not a local maximum of f.

5) We have a minimum at {?, -3.55122, -0.3} and {3, 2, 0} is not a saddle point of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $7x + 4z == 9$ and the paraboloid $z == 8x^2 + 8y^2$.

1) 1.08038

2) 2.91641

3) 0.107038

4) 2.71195

5) 0.27659

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {2 Cos[u] Sin[v] - 3 Sin[u] Sin[v], -4 Cos[v] - 4 Cos[u] Sin[v] + 12 Sin[u] Sin[v], 4 Cos[v] + 6 Cos[u] Sin[v] - 12 Sin[u] Sin[v]} .

1) The maximum Gauss curvature is **1.****

2) The maximum Gauss curvature is **7.****

3) The maximum Gauss curvature is **2.****

4) The maximum Gauss curvature is **3.****

5) The maximum Gauss curvature is **8.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]---\to R^2$

$r(t)=\{(9t+2)\sin(2t)(7\cos(11t)+9),(5t+8)\sin(t)(7\cos(11t)+9)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 60 000.9 2) 3750.9 3) 33 750.9 4) 37 500.9

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -10x & 0\le x\le \frac{2}{5} \\ 2x-\frac{24}{5} & \frac{2}{5}\le x\le \frac{9}{10} \\ 30x-30 & \frac{9}{10}\le x\le 1 \end{cases} & 0\le x\le 1 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} \frac{35x}{2} & 0\le x\le \frac{2}{5} \\ 10x+3 & \frac{2}{5}\le x\le \frac{3}{5} \\ \frac{45}{2}-\frac{45x}{2} & \frac{3}{5}\le x\le 1 \end{cases} & 0.\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=\frac{4}{5}$
and the moment t= 0.005 by means of a Fourier series of order 12 .

1) $u(\frac{4}{5}, 0.005) =$ **4.****

2) $u(\frac{4}{5}, 0.005) =$ **0.****

3) $u(\frac{4}{5}, 0.005) =$ **3.****

4) $u(\frac{4}{5}, 0.005) =$ **6.****

5) $u(\frac{4}{5}, 0.005) =$ **2.****

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Exercise 1

Given the function

$f(x, y, z) = 4 - 6x + x^2 + 2y - y^2 - 2z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-2.62191, ?, -3.19506} and {3, 1, 1} is not a local minimum of f.

2) We have a maximum at {-2.42191, ?, -2.49506} and {3, 1, 1} is not a local minimum of f.

3) We have a maximum at {?, 1, 1} and {3, 1, 1} is not a saddle point of f.

4) We have a maximum at {?, 0.535052, -2.79506} and {3, 1, 1} is not a local minimum of f.

5) We have a maximum at {?, 0.135052, -3.09506} and {3, 1, 1} is not a local minimum of f.

6) None of the other answers is correct

Exercise 2

Compute the volume of the domain limited by the plane $8x + 6z == 10$ and the paraboloid $z == 8x^2 + 8y^2$.

1) 0.376099

2) 0.229433

3) 0.582382

4) 2.5111

5) 1.93339

Exercise 3

Compute the maximum value of the Gauss curvature for X(u,v) = {Cos[u] (2 + Cos[v]) - (2 + Cos[v]) Sin[u], (2 + Cos[v]) Sin[u], Cos[u] (2 + Cos[v]) - (2 + Cos[v]) Sin[u] + Sin[v]}.

1) The maximum Gauss curvature is **6.****

2) The maximum Gauss curvature is **0.****

3) The maximum Gauss curvature is **2.****

4) The maximum Gauss curvature is **8.****

5) The maximum Gauss curvature is **4.****

Exercise 4

Compute the area of the domain whose boundary is the curve

$r:[0,\pi]\dashrightarrow R^2$
$r(t)=\{(8t+6)\sin(2t)(6\cos(18t)+6),\ (5t+4)\sin(t)(6\cos(18t)+6)\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 31 453.6   2) 1656.39   3) 16 555.   4) 14 899.6

Exercise 5

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t)=16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t)=u(3,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -5x & 0\le x\le 1 \\ 7x-12 & 1\le x\le 2 \\ 6-2x & 2\le x\le 3 \end{cases} & 0\le x\le 3 \\ \frac{\partial}{\partial t}u(x,0)=\begin{cases} 5x & 0\le x\le 1 \\ 14-9x & 1\le x\le 2 \\ 4x-12 & 2\le x\le 3 \end{cases} & 0.\le x\le 3 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x = 1
and the moment t = 0.003 by means of a Fourier series of order 12 .

1) u(1 , 0.003) = **4.****

2) u(1 , 0.003) = **2.****

3) u(1 , 0.003) = **9.****

4) u(1 , 0.003) = **3.****

5) u(1 , 0.003) = **5.****

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Exercise 1

Given the system

$2 v^3 + 3 v w x + w^2 x - 3 w x^2 - u y - 3 u v y == -66$
$1 - u^2 v + 3 v^2 x + x^2 + 3 y + v w y - w^2 y == 61$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p = (x , y , u , v , w) = (0 , -2 , 5 , -2 , -4). Compute if possible the approximate values of (x , y) for (u , v , w) = (4.9 , -1.7 , -4.2) by means of the tangent affine function at p.

1) (x , y) ≃ (0.463 , -2.8888)

2) (x , y) ≃ (0.363 , -2.5888)

3) (x , y) ≃ (0.063 , -2.7888)

4) (x , y) ≃ (0.363 , -2.4888)

5) (x , y) ≃ (-0.037 , -2.2888)

Exercise 2

Compute the volume of the domain limited by the plane $6 x + 4 z == 8$ and the paraboloid $z == 6 x^2 + y^2$ and the semiplanes $9 x + 5 y \geq 0$ and $7 x - 4 y \geq 0$.

1) 0.304704

2) 1.28899

3) 0.219358

4) 0.230366

5) 0.127442

Exercise 3

Compute the maximum value of the Gauss curvature for $X(u,v) = \{(1 + 2 v^2) \text{Cos}[u], -v + 2 (1 + 2 v^2) \text{Cos}[u] + (1 + 2 v^2) \text{Sin}[u], 2 v - (1 + 2 v^2) \text{Cos}[u] - (1 + 2 v^2) \text{Sin}[u]\}$.

1) The maximum Gauss curvature is **3.****

2) The maximum Gauss curvature is **4.****

3) The maximum Gauss curvature is **0.****

4) The maximum Gauss curvature is **7.****

5) The maximum Gauss curvature is **2.****

Exercise 4

Consider the vector field $F(x,y,z) = \{5y + \text{Cos}[y^2 + 2z^2],\ e^{2x^2 - z^2} + 4y,\ 8y - \text{Sin}[2x^2 + 2y^2]\}$ and the surface

$$S \equiv \left(\frac{-8+x}{9}\right)^2 + \left(\frac{y}{2}\right)^2 + \left(\frac{-5+z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 5306.77 2) 1206.37 3) 0.371579 4) −2049.83

Exercise 5

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = -3(x-2)(x-1)x^2(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.003$ by means of a Fourier series of order 8 .

1) u(1 , 0.003) = ***.**5*

2) u(1 , 0.003) = ***.**4*

3) u(1 , 0.003) = ***.**8*

4) u(1 , 0.003) = ***.**3*

5) u(1 , 0.003) = ***.**2*